MULLARD LIMITED
Mullard House
Torrington Place
London, W.C.1

CONTENTS

Thyristor Speed Control of D.C. Shunt Motors from a Single Phase Supply 298–315

Instructions for Selecting a D.C. Motor for Thyristor Speed Control 316–327

Articles on Power Topics 328

VOL. 8 No. 80 MARCH 1966

Mullard Technical Communications

VOLUME 8 NUMBER 80

March 1966

Mullard

Mullard Technical Communications

VOLUME 8 NUMBER 80 March 1966

Contents

Thyristor Speed Control of D.C. Shunt Motors from a Single Phase Supply *J. Merrett* 298

Instructions for Selecting a D.C. Motor for Thyristor Speed Control *J. Merrett* 316

Articles on Power Topics 328

Mullard Technical Communications contains information on the performance or application of Mullard products such as electronic valves and tubes, semiconductor devices, and electrical and magnetic components. It is issued free-of-charge to equipment designers, research organisations, reference libraries, and educational establishments. Bona fide applications to be added to the circulation list should be made under an official letter head to Central Technical Services. Overseas enquiries should be addressed to the local Mullard agent, or to Mullard Export Division at Mullard House.

Abstracts, or references to an article, should mention both the source, *Mullard Technical Communications*, and the authors. Requests for republication or translation should be made to the Editor in writing.

The issue of the information contained in this publication does not imply any authority or licence for the utilisation of any patented feature.

'Mullard' is the trade mark of Mullard Limited and is registered in most of the principal countries of the world.

Editor: F. A. Sowan

Assistant Editors: R. Barrie E. G. Evans

Published by

Central Technical Services

Mullard Limited, Mullard House, Torrington Place, London, W.C.1

LANgham 6633

© Mullard Limited, 1966

Made and Printed in England by

WIGHTMAN AND COMPANY LIMITED, 1/3 BRIXTON ROAD LONDON, S.W.9

Thyristor Speed Control of D.C. Shunt Motors from a Single Phase Supply

J. MERRETT
Mullard Central Application Laboratory

Thyristors are considered in a power control system for a d.c. shunt-wound motor. Graphs enable the selection of motors suitable to meet given specifications, and indicate the performance to be expected under various operating conditions.

An appended series of instructions enable a motor to be selected to meet any stated requirements, and a worked example illustrates the use of these examples.

INTRODUCTION

The speed control of d.c. motors with thyratrons and thyristors has been quite well documented from the point of view of the circuit designer, but much less has been written about the integration of the control circuits with actual motors, and about the overall performance to be expected from the motor. An attempt is made in this article to compare the mechanical performance of thyristor controlled d.c. shunt motors with the known performance of these motors when operated from d.c. supplies, and to outline a method by which a suitable motor can be selected for a particular application, when the mechanical requirements are known. Once the correct motor has been chosen the electrical requirements of the control system can be calculated. This article is confined to systems supplied from single phase a.c. mains, and deals only with phase control systems. Three phase and d.c. chopper type controls are outside the scope of the present article, but some of the conclusions reached will be applicable to these other control systems.

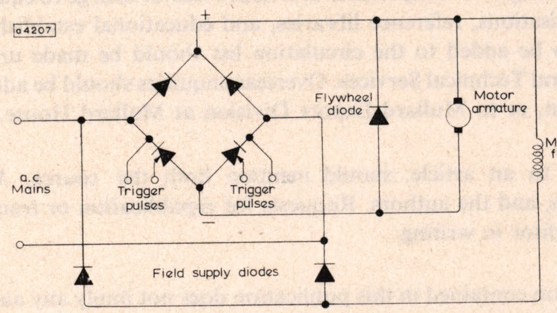


Fig. 1—Half-controlled thyristor bridge, with flywheel diode and field supply

ELEMENTARY THYRISTOR CONTROL CIRCUITS

Only two basic circuits need be considered; these are shown in Fig. 1 and Fig. 2. The circuit in Fig. 1 shows a normal half-controlled bridge with a flywheel diode and additional subsidiary rectifiers for the field supply, while Fig. 2 shows a circuit using only one thyristor. Both circuits produce identical waveforms as far as the motor is concerned. As the choice of circuit has no bearing upon the motor analysis, the factors influencing this choice are not dealt with in this article.

MOTOR ANALYSIS WITH THYRISTOR CONTROLLED SUPPLY

To simplify the solution of the motor equations in the present case where the motor is supplied with unidirectional current pulses instead of a steady current, a simplified motor model will be adopted. The following simplifying assumptions will be made:

(a) The field provides a constant steady air gap flux Φ.

(b) The armature current does not produce sufficient reaction field to influence the air gap flux.

(c) The armature flux is directly proportional to armature current I, and there are no iron-saturation effects.

(d) Armature resistance R is constant and embodies all armature 'copper loss' effects.

(e) All other motor losses can be combined as a variable loss which is directly proportional to speed.

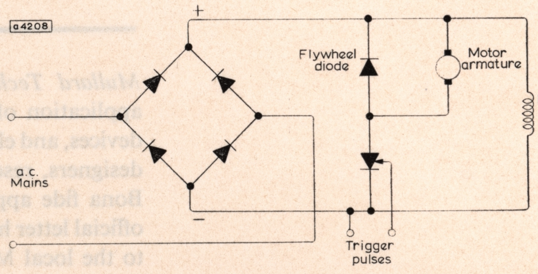


Fig. 2—Full-wave bridge with single thyristor and flywheel diode

The following motor equations can then be written down using the symbols listed in Appendix 1.

$$\text{Speed, } n = (E-IR)/C_1\Phi \qquad \ldots(1)$$

$$\text{Torque, } T = C_2.I.\Phi \qquad \ldots(2)$$

$$\text{Power Output, } P_o = C_3.n.T \qquad \ldots(3)$$

$$\text{Power Loss, } P_l = I^2.R + C_4.n \qquad \ldots(4)$$

$$\text{Efficiency (armature only), } \eta = P_o/(P_o+P_l) \qquad \ldots(5)$$

where C_1 to C_4 are constants

The power required to energise the field has been neglected.

From these equations, further equations can be derived which can be used as a basis for the analysis of the motor when supplied with a phase controlled voltage supply.

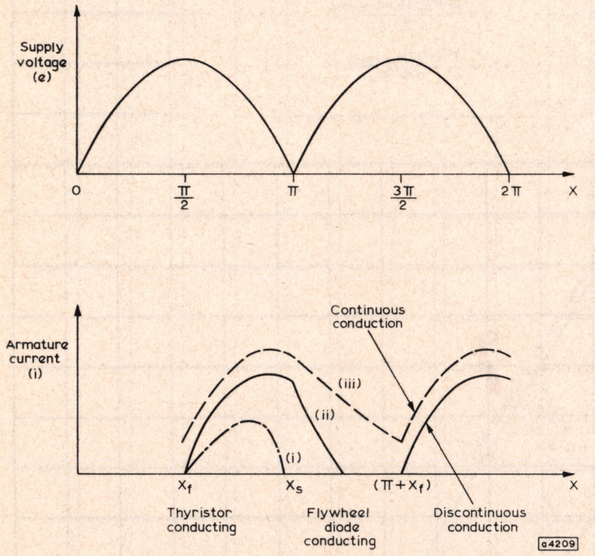


Fig. 3—Current waveforms for armature with flywheel diode

Before proceeding with the circuit analysis it will be helpful to outline the operation of the circuit. Fig. 3 indicates the three possible types of current waveform. These are described below.

(i) The thyristor is turned on at some phase angle x_f and turned off at angle x_s, where $x_s < \pi$. This condition is obtained when motor speed is high, giving a high back e.m.f., and armature inductive back e.m.f. (L.di/dt) is low. This occurs with a low-torque load. The flywheel diode does not conduct at all under these conditions.

(ii) The thyristor is turned on at x_f and turned off at $x_s = \pi$, when the flywheel diode starts to conduct. This condition applies for moderate motor speeds and loads; the armature current is then discontinuous.

(iii) The thyristor is turned on at x_f and turned off at $x_s = \pi$, but the flywheel diode is still conducting when the thyristor is refired at x_f in the next half-cycle. This condition applies for low-speed high-torque loads, and the armature current is continuous.

The motor armature current equations will be different in these three cases, so before the analysis can proceed far, it is necessary to ascertain the type of current waveform which is applicable.

For the circuit of Fig. 1, but without a flywheel diode, the equation for instantaneous armature current can be shown to be as follows:

$$i = \frac{\sqrt{2}E_s}{R}\left\{\cos\phi.\sin(x-\phi)-a+\right.$$

$$\left.+[a-\cos\phi.\sin(x_f-\phi)]\exp\left[\frac{-(x-x_f)}{\tan\phi}\right]\right\} \qquad \ldots(6)$$

where the supply voltage is given by:

$$e = \sqrt{2}E_s.\sin x \qquad \ldots(7)$$

and

$$\text{Armature, } \tan\phi = \frac{\omega L}{R} \qquad \ldots(8)$$

$$\text{Speed factor, } a = \frac{V}{\sqrt{2}E_s}. \qquad \ldots(9)$$

If Eq.6 is solved for $i = 0$, a value of x will be obtained at which the armature current, or thyristor current, has fallen to zero. If the solution yields a value of $x_s < \pi$ the circuit operation will be type (i) and the integral of i between limit $x = x_f$ and $x = x_s$ will give the average armature and thyristor current. The flywheel diode, if one was present, would not conduct.

If the solution gives a value of $x_s > \pi$, then a flywheel diode, if present, will conduct and turn off the thyristor at $x_s = \pi$. It should be noted that the circuit of Fig. 2 requires a flywheel diode to turn off the thyristor in all cases when $x_s > \pi$.

The present article is only concerned with circuits using flywheel diodes, so in the case of $x_s > \pi$, Eq.6 will not give the correct solution.

In those cases where the flywheel diode conducts, the thyristor current is transferred to the diode at the end of the cycle. From this point the motor armature is disconnected from the supply and armature current decays exponentially through the diode at a rate dependent upon the time constant (L/R) of the armature and the generated back e.m.f. (V) caused by rotation. A high L/R increases the conduction time and a high V decreases the conduction time. From the equation for diode current the conduction time and average current value can be calculated.

If, in Fig. 3, the diode conduction time is less than x_f, armature current is discontinuous and is of type (ii). In this case Eq.6 can be integrated between the limits $x = x_f$ and $x = \pi$ to give the average value of thyristor current. The average armature current is then obtained by summing average thyristor and flywheel diode currents.

If the solution of the flywheel diode current equations shows that the diode is still conducting at angle $(\pi+x_f)$ when the thyristor is refired, armature current is continuous and of type (iii). A further modification has to be made to the equations in this case to allow for this steady component of current.

It was stated earlier that the firing angle x_f is determined by the thyristor trigger circuit. While this statement is, in general, true, in practice a thyristor cannot be fired until a positive voltage exists between anode and cathode. When the motor is rotating it generates a back e.m.f. proportional to speed. This provides a reverse bias for the thyristor. The thyristor will only have a forward

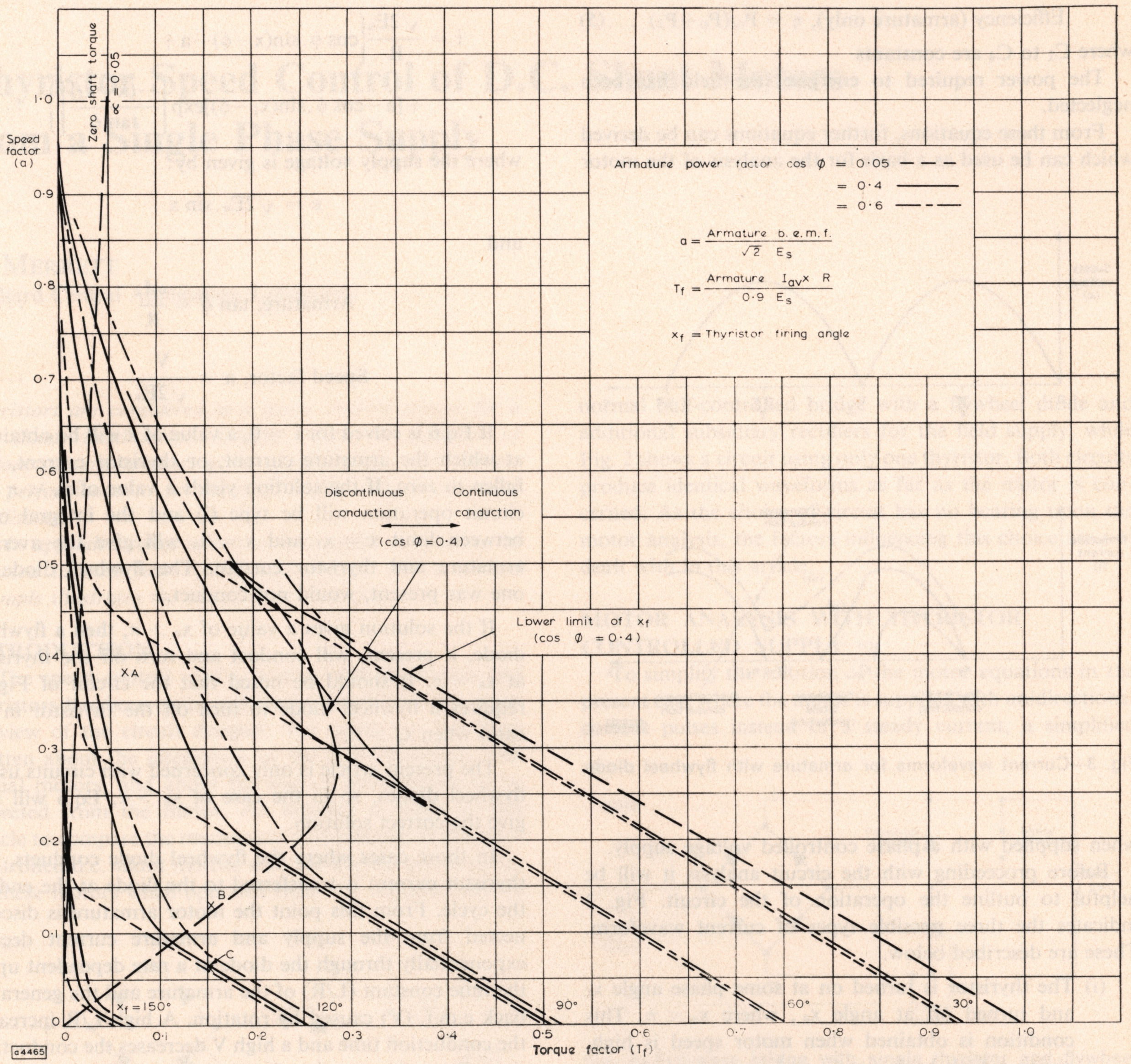


Fig. 4—Normalised speed-torque curves for shunt motor, with thyristor firing angle as parameter

voltage across it, and can therefore only be turned on, under the following conditions:

$$x_1 < x_f < x_2 \qquad \ldots(10)$$

where

$$x_1 = \sin^{-1}a \qquad \ldots(11)$$

$$x_2 = \pi - \sin^{-1}a. \qquad \ldots(12)$$

The restrictions imposed by these equations must be borne in mind when selecting the firing angle x_f at which the trigger circuit is to operate, or when computing ranges of possible operating conditions.

The equations for thyristor and armature average current become lengthy for all operating conditions except type (i). They are given in detail in Appendix 2. The solutions to these equations for a range of speed and firing angles have been calculated with the aid of a computer and some of the results are presented in graphical form in Figs. 4 and 5. The effect of changes in armature power factor are shown.

IMPLICATIONS OF THE ANALYSIS

The results of the analysis presented in Figs. 4 and 5 have been reduced to a normalised form where the factors involved have been defined as follows:

$$\text{Speed Factor, } a = \frac{V}{\sqrt{2}E_s} \qquad \ldots(13)$$

$$\text{Torque Factor, } T_f = \frac{\text{Torque}}{\text{Max Stalled Torque}} \qquad \ldots(14)$$

or

$$T_f = \frac{T.R}{C_2.E.\Phi}. \qquad \ldots(15)$$

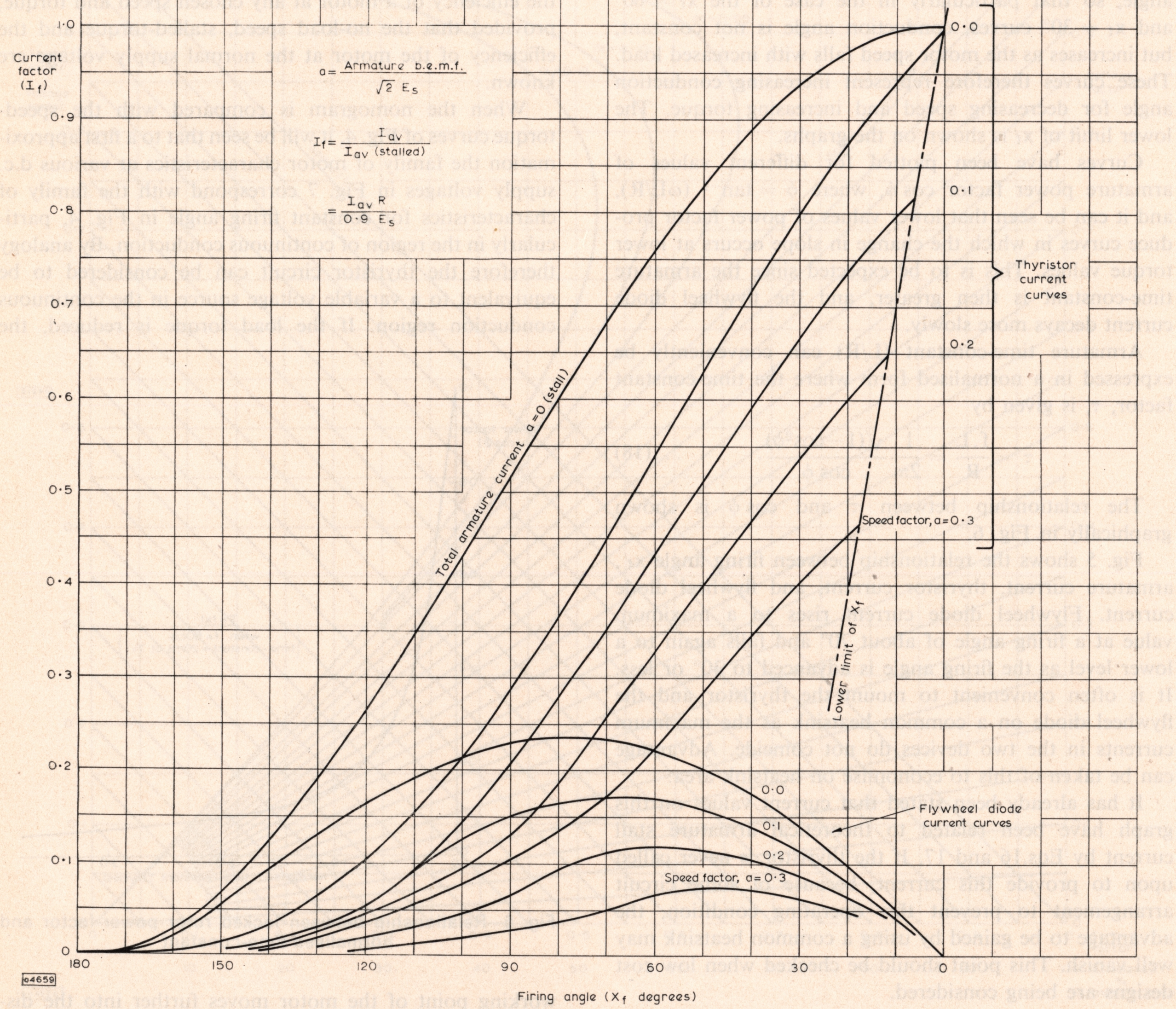


Fig. 5—Normalised thyristor and flywheel diode average-current firing angle curves, with speed-factor as parameter (Armature power factor, cos φ = 0·4)

Currents are expressed in terms of a current factor where,

$$I_f = \frac{I_{av}}{\text{Max Armature Stalled Current } (x_f = 0)} \qquad \ldots(16)$$

or

$$I_f = \frac{I_{av}.R}{0{\cdot}9E_s}. \qquad \ldots(17)$$

In all cases firing angle x_f has been used as a parameter in preference to the more usual conduction angle, since, in motor circuits, the conduction period of the thyristor may be a function of both motor speed and firing angle. Conduction angle θ_c becomes $(\pi - x_f)$ only when the motor speed is zero, or when the inductive back e.m.f. of the armature is sufficient to maintain conduction to the end of the cycle in the face of a generated e.m.f., which is opposing the supply voltage and attempting to turn the thyristor off. In general θ_c is less than $(\pi - x_f)$: this occurs at high speed and low torque when armature current, and therefore L.di/dt of the armature, is low. All conventional thyristor trigger modules are designed to control firing angle; the thyristor turn-off angle is determined by the point at which the voltage across the thyristor reverses. It can be seen from Fig. 4 that the motor speed-torque curves have two distinct parts, one of high slope and another with a slope very similar to that of the machine being run from an equivalent d.c. supply. The first corresponds with the discontinuous armature current condition and the second with the continuous conduction, or nearly d.c. condition. As the armature current becomes more nearly d.c. the slope of the speed-torque curve tends to the d.c. slope.

It should be remembered that these curves are plotted for constant firing angle, and not constant conduction angle, so that particularly in the case of the $x_f = 60°$ and $x_f = 30°$ curves, conduction angle is not constant, but increases as the motor speed falls with increased load. These curves therefore represent increasing conduction angle for decreasing speed and increasing torque. The lower limit of x_f is shown on the graphs.

Curves have been plotted for different values of armature power factor $\cos\phi$, where $\phi = \tan^{-1}(\omega L/R)$, and it can be seen that lower values of power factor produce curves in which the change in slope occurs at lower torque values. This is to be expected since the armature time-constant is then greater, and the flywheel diode current decays more slowly.

Armature time-constant (L/R) can conveniently be expressed in a normalised form where the time-constant factor, τ, is given by

$$\tau = \frac{f.L}{R} = \frac{1}{2\pi}\cdot\frac{\sqrt{(1-\cos^2\phi)}}{\cos\phi}. \qquad \ldots(18)$$

The relationship between τ and $\cos\phi$ is shown graphically in Fig. 6.

Fig. 5 shows the relationship between firing angle x_f, armature current, thyristor current, and flywheel diode current. Flywheel diode current rises to a maximum value at a firing angle of about 70° and falls again to a lower level as the firing angle is advanced to 30° or less. It is often convenient to mount the thyristor and the flywheel diode on a common heatsink as the maximum currents in the two devices do not coincide. Advantage can be taken of this to economise on heatsink area.

It has already been stated that current values on this graph have been related to theoretical armature stall current by Eqs.16 and 17. If the thyristor is never called upon to provide this current, because of some circuit arrangement to prevent this operating condition, the advantage to be gained by using a common heatsink may well vanish. This point should be checked when low cost designs are being considered.

Examination of the slope of the speed-torque curves in Fig. 4 shows that speed regulation of the motor will be much worse than for the d.c. machine in the region of discontinuous conduction. This difficulty can be minimised by arranging that the motor working region falls within the region of continuous conduction, either by working at relatively high torque, or by increasing the motor armature time-constant to increase the region of continuous conduction. Unfortunately neither of these techniques can be applied to a sufficient degree to make a great deal of difference to the practical working region of a machine. The reason for this will be discussed below.

SELECTION OF A SUITABLE MOTOR FROM A MECHANICAL SPECIFICATION

Before considering in detail the special case of a thyristor controlled motor, it will be helpful to take the simple case of a d.c. shunt motor with a variable voltage armature supply.

If the simplified machine model, defined earlier in this article, is assumed, a nomogram (Fig. 7) can be constructed to relate speed, torque and efficiency for a range of armature voltages. This nomogram can then be used to obtain the efficiency of a motor at any chosen speed and torque, provided that the no-load speed, stalled-torque and the efficiency of the motor at the normal supply voltage are known.

When the nomogram is compared with the speed-torque curves of Fig. 4, it will be seen that to a first approximation the family of motor characteristics at various d.c. supply voltages in Fig. 7 correspond with the family of characteristics for constant firing angle in Fig. 4, particularly in the region of continuous conduction. By analogy therefore the thyristor circuit can be considered to be equivalent to a variable voltage source in the continuous conduction region. If the load torque is reduced, the working point of the motor moves further into the discontinuous conduction region and the operating characteristics of the thyristor controlled motor deviate more and more from the d.c. drive characteristics, finally approaching the condition of constant current drive. The nomogram Fig. 7 will therefore give results that are approximately true for thyristor drives only if the armature current is continuous, that is, approximating to true d.c. In most cases motor efficiencies will need to be known only under full load conditions, as it is under these conditions that maximum armature heating effects are troublesome and maximum power consumption occurs. The fact that the nomogram will give very inaccurate results under low load conditions is therefore not a very serious limitation.

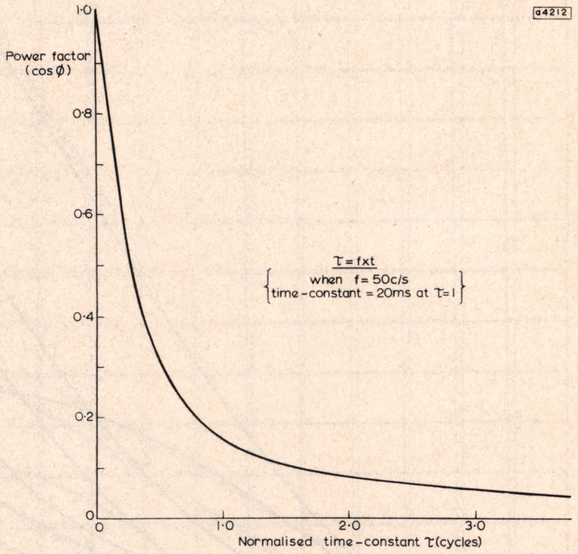


Fig. 6—Relationship between locked rotor power-factor and normalised time-constant

Correction may be applied to the results obtained from the nomogram to allow for the errors introduced by the departure of the armature current waveform from true d.c. For the case of a d.c. shunt motor, supplied with true d.c., torque is proportional to armature current and armature copper loss is proportional to the square of this current. When the same motor is supplied from a thyristor circuit

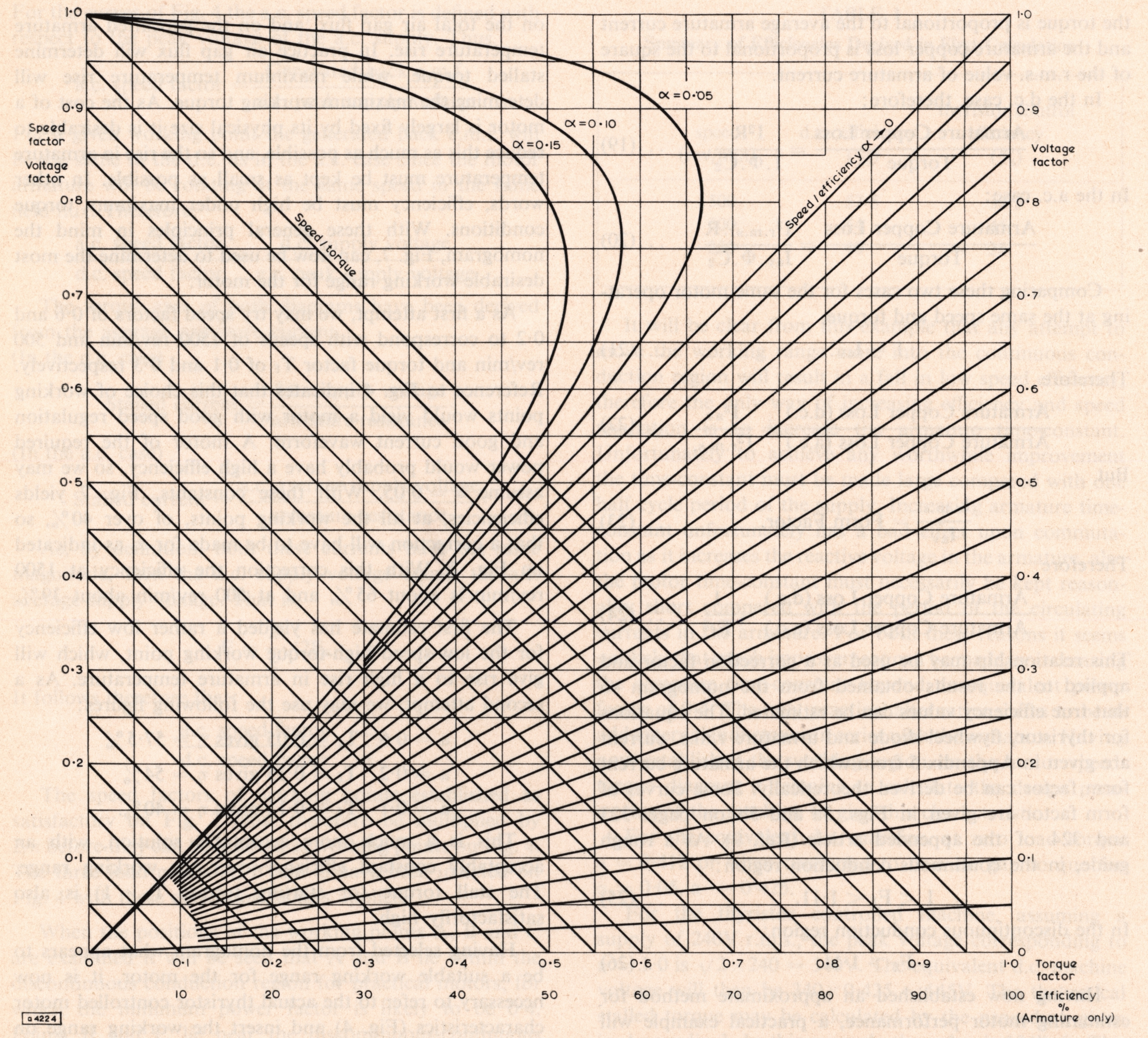
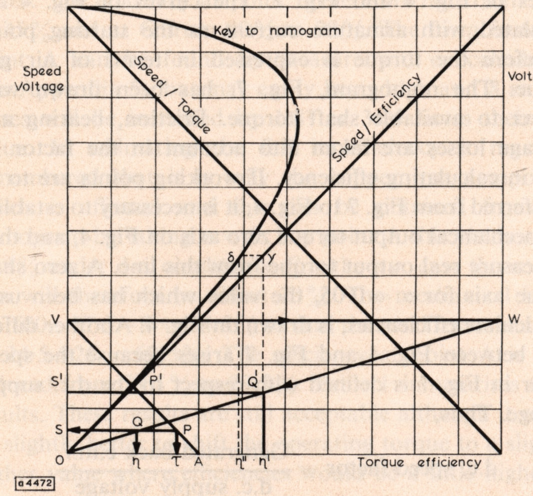


Fig. 7—Nomogram relating speed, torque and efficiency for d.c. shunt motor with armature voltage speed-control

To find motor efficiency when operating speed and torque are known

1. Select a speed-torque line which passes through the desired operating point P corresponding with speed S and torque T, line VPA.
2. Select a speed-efficiency line WQO having the same voltage-factor as line VPA. The intersection of this line with the speed line SQP, gives the required motor efficiency η.
3. The above method gives η for $\alpha = 0$. This is approximately correct if $\eta < 40\%$. For higher values of η, the efficiency can be found in the following way.
   First evaluate the efficiency for $\alpha = 0$, as already explained. Find the point of intersection between this efficiency line (say η') and the speed-efficiency line for $\alpha = 0$. (This would give the point γ.) Draw a horizontal line through this point to intersect the speed-efficiency curve for the correct value of α. (This would give the point δ.) The new efficiency (η'') can then be read off from the abscissa below this point (V).

   N.B.—Air gap torque at no load is assumed to be proportional to speed, and includes all losses (bearing, windage, iron, etc.) except copper losses. Field losses and armature reaction are not taken into account.



the torque is proportional to the average armature current and the armature copper loss is proportional to the square of the r.m.s. value of armature current.

In the d.c. case, therefore:

$$\frac{\text{Armature Copper Loss}}{\text{Torque}} = \frac{I^2R}{I.\Phi.C_2}. \qquad \ldots(19)$$

In the a.c. case:

$$\frac{\text{Armature Copper Loss}}{\text{Torque}} = \frac{I^2_{\text{r.m.s.}}.R}{I_{\text{av}}.\Phi.C_2}. \qquad \ldots(20)$$

Comparing these two cases for the same motor operating at the same speed and torque,

$$I = I_{\text{av}}. \qquad \ldots(21)$$

Therefore

$$\frac{\text{Armature Copper Loss (d.c.)}}{\text{Armature Copper Loss (a.c.)}} = \frac{I^2_{\text{av}}}{I^2_{\text{r.m.s.}}}. \qquad \ldots(22)$$

But

$$\frac{I_{\text{r.m.s.}}}{I_{\text{av}}} = \text{Form Factor}. \qquad \ldots(23)$$

Therefore

$$\frac{\text{Armature Copper Loss (d.c.)}}{\text{Armature Copper Loss (a.c.)}} = \frac{1}{F_f^2} \qquad \ldots(24)$$

This relationship may be used as a correction factor and applied to the results obtained from the nomogram so that true efficiency values can be estimated. The equations for thyristor, flywheel diode and armature r.m.s. current are given in Appendix 5 from which the armature current form factor can be derived theoretically. Some curves of form factor are given in Figs. 5a and 5b on pages 323 and 324 of the appended article (Ref. 9). As a rough guide, in the continuous conduction region:

$$1 < F_f < 1{\cdot}11. \qquad \ldots(25)$$

In the discontinuous conduction region:

$$F_f > 1{\cdot}11. \qquad \ldots(26)$$

Having now established an approximate method for estimating motor performance, a practical example will be considered to illustrate how this information can be used to select a suitable motor for a particular application.

Suppose that the mechanical specification demands a motor which is normally required to run between speeds of 1500 rev/min and 500 rev/min and sometimes for short periods at 250 rev/min. The maximum continuous torque at 1500 rev/min is 10 lb.ft rising with fall in speed to 30 lb.ft at 500 rev/min. The torque at 250 rev/min would not exceed 30 lb.ft.

First observations are that between 500 rev/min and 1500 rev/min the motor is required to deliver constant horse power (h.p. = 2·85). At 250 rev/min the required power output is lower and the torque does not exceed the maximum at 500 rev/min. The duty cycle is also low, so this working condition can be considered at a later stage in the design. Starting torque has not been specified so it can be assumed to be unimportant, provided, of course, that it is greater than the specified maximum torques required.

The physical size of the motor will be determined by the maximum torque required, which will set a lower limit on the total air gap flux, and on the permitted armature temperature rise. In practice air gap flux will determine stalled torque, while maximum temperature rise will determine the maximum working torque. As the cost of a motor is largely fixed by its physical size it is desirable to reduce this as much as possible, and so the rise in armature temperature must be kept as small as possible. In other words, efficiency must be high under maximum torque conditions. With these general principles in mind the nomogram, Fig. 7, can now be used to determine the most desirable working range for the motor.

As a first attempt, we may try speed factors of 0·6 and 0·2 to correspond with speeds of 1500 rev/min and 500 rev/min and torque factor T_f of 0·1 and 0·3 respectively. Reference to Fig. 4 indicates that this choice of working points would yield a motor with good speed regulation and good current waveform. A motor of the required power would probably have a high efficiency, so we may assume $\alpha = 0{\cdot}05$. With these constants, Fig. 7 yields efficiencies, at all the working points, of over 40%, so that a correction will have to be made for α, as indicated on Fig. 7. With this correction the efficiency at 1500 rev/min is about 65%, and at 500 rev/min about 39%.

The first estimate has yielded a rather low efficiency for the low-speed high-torque working point, which will give rise to a high rise in armature temperature. As a second attempt, we may use the following figures:

$$a = 0{\cdot}6 \quad T_f = 0{\cdot}05 \text{ gives } \eta = 57{\cdot}5\%$$

$$a = 0{\cdot}2 \quad T_f = 0{\cdot}15 \text{ gives } \eta = 54\%$$

$$a = 0{\cdot}1 \quad T_f = 0{\cdot}15 \text{ gives } \eta = 40\%.$$

This is a much more acceptable solution, with an acceptably constant efficiency over the working range. The stall torque (or starting torque $T_f = 1$) is also satisfactorily high.

Having selected from the nomogram what appears to be a suitable working range for the motor, it is now necessary to refer to the actual thyristor controlled motor characteristics (Fig. 4) and insert the working range on this graph.

At this point it is necessary to digress slightly to point out a significant difference between the speed-torque curves of Fig. 4 and Fig. 7. The curves of Fig. 4 are calculated with armature current as the starting point. Therefore the torque is expressed in terms of air gap torque. The nomogram, Fig. 7, has been drawn with respect to available shaft torque. Friction, bearing and windage losses are taken into account in the factor 'α' used in calculating efficiency. If working points are to be transferred from Fig. 7 to Fig. 4, it is necessary to establish the mechanical output torque zero axis on Fig. 4, and then to measure real output torque from this line. A zero shaft torque axis for $\alpha = 0{\cdot}05$, the value which has been used to calculate efficiencies, is drawn on Fig. 4. Another difference between Fig. 4 and Fig. 7 arises because the speed factor in Fig. 7 is defined with respect to the d.c. supply voltage. Thus,

$$\text{d.c. speed factor} = \frac{\text{Generated back e.m.f.}}{\text{d.c. supply voltage}}$$

For the curves of Fig. 4 the a.c. speed factor is defined with respect to the peak voltage of the a.c. supply. Thus,

$$\text{a.c. speed factor} = \frac{\text{Generated back e.m.f.}}{\text{a.c. peak supply voltage}}$$

For a given working point the generated back e.m.f. will be the same in the d.c. as in the a.c. case since the motor armature will be rotating at the same speed in the same field flux. Therefore,

$$\frac{\text{a.c. speed factor}}{\text{d.c. speed factor}} = \frac{\text{d.c. supply voltage}}{\text{a.c. peak supply voltage}}.$$

The torque axes of both diagrams have been derived from the average armature current,
(In the d.c. case)

$$\text{Stalled current} = \frac{\text{d.c. supply voltage}}{\text{armature resistance}}.$$

(In the a.c. case)

$$\text{Stalled current} = \frac{\text{a.c. average supply voltage}}{\text{armature resistance}}.$$

As, by definition, both these currents have the same magnitude, then the d.c. supply voltage equals the a.c. average supply voltage. That is,

$$E = \frac{2}{\pi}\sqrt{2}E_s.$$

It follows therefore that:

$$\frac{\text{a.c. speed factor}}{\text{d.c. speed factor}} = \frac{2}{\pi}.$$

The speed factors of working points established as satisfactory on Fig. 7 must therefore be multiplied by $2/\pi$ before they can be inserted as working points on Fig. 4. The three load points A, B and C have been inserted on Fig. 4 with these correction factors applied.

When the positions of the working points A, B and C are examined it will be seen that A and B lie within the discontinuous conduction region for practical motors, for which the minimum power factor is likely to be 0·4. Points B and C lie near the discontinuous-continuous boundary where armature current form factor will be about 1·1. The losses in this region will therefore be increased by about $1{\cdot}1^2$ or 1·2 times the equivalent d.c. losses. The efficiency at point B therefore falls from 54% to 45% and at C from 40% to 28%. The form factor at point A can be found by measurement or calculation (see Appendix 5), but may easily be as high as 1·8 so that losses will be increased by $1{\cdot}8^2$ or 3·25 times their d.c. value. In estimating the fall in efficiency it must be remembered that it is only the copper loss component which is affected and this can be obtained from the nomogram by taking $\alpha = 0$. For point A this yields a loss of 8% under d.c. conditions. For the a.c. case the copper loss will therefore be about 26%. The mechanical losses are $100-57{\cdot}5-8 = 34{\cdot}5\%$ so that the total losses amount to $34{\cdot}5+26 = 60{\cdot}5\%$. The efficiency at point A is thus reduced from 57·5% to 39·5%. Table 1 summarises these results. These results are still acceptable although it might be slightly better to shift the operating point to a slightly higher value where efficiencies would be a little higher.

TABLE 1

Summary of Efficiencies Derived in First Calculation

speed (rev/min)	d.c. efficiency (%)	thyristor control efficiency (%)
1500	57·5	39·5
500	54	45
250	40	28

It will be clear from this example that any attempt to move the working range more into the continuous conduction region will result in a fall in low speed efficiency, therefore the only way of increasing efficiency and speed regulation is to increase the armature time-constant. Unfortunately to achieve any worthwhile improvement the time-constant must be made large compared with one half cycle period of the supply. Increasing armature time-constant unfortunately has a bad effect upon commutation as it increases the reactive voltage in the armature, also the motor time-constant must necessarily be kept reasonably short compared with the period of the circulating currents in the armature. For both these reasons it seems unlikely that much improvement can be made by increasing the armature time-constant beyond an equivalent power factor of 0·4. The only practical alternative seems to be to add external inductance in series with the armature. Having now established the working region on the motor speed-torque diagram it remains to calculate the design parameters of the machine to give this characteristic.

For the equivalent d.c. machine, at zero load torque $a = 0{\cdot}412$ or when converted to true air gap torque for $\alpha = 0{\cdot}05$, $a = 0{\cdot}425$.

For the thyristor controlled machine, assuming a supply of 240V r.m.s., the peak voltage corresponding to $a = 1{\cdot}0$ is $\sqrt{2} \times 240 = 340$V. The equivalent d.c. machine voltage will then be $340 \times 0{\cdot}425 = 145$V. The theoretical stalled torque may be calculated by the procedure given in Appendix 3. In this example the value is 134lb.ft. This data will enable the motor designer to establish the motor design, although it must be remembered that the actual stalled-torque will probably be rather less than this theoretical value because of armature reaction and other causes. The theoretical value of stalled-torque must be used to determine the magnitude of armature resistance, as this fixes the motor working region on the speed-torque diagram.

The steps taken in this worked example are summarised below:

(i) The nomogram Fig. 7 is used to determine a motor working region so that the motor efficiency is reasonably high over the speed and torque range.

(ii) The working range selected from the nomogram is transferred to the thyristor speed-torque diagram, Fig. 4.

(iii) The efficiency obtained from the nomogram is corrected, if necessary, by using the armature current form factor relationship in Eq.24.

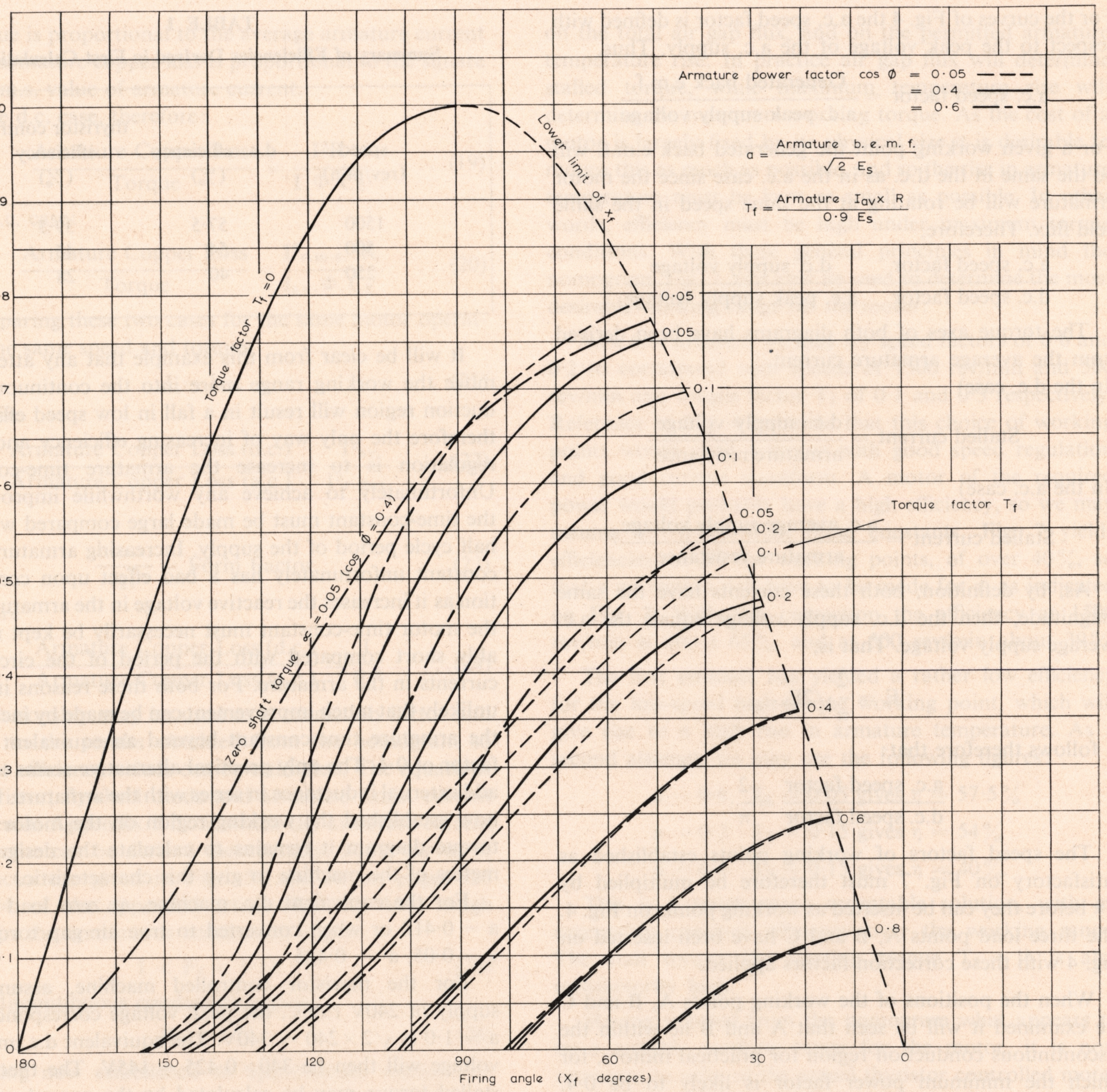


Fig. 8—Normalised speed versus firing angle curves for shunt motor, with torque factor as parameter

- (iv) The working region may have to be changed slightly if efficiencies are appreciably reduced by application of Eq.24.
- (v) Having established the optimum working region of the motor, the true d.c. speed factor at zero air gap torque is obtained from the graph, Fig. 4. The peak voltage of the a.c. supply to be used is then multiplied by this speed factor to obtain the d.c. supply voltage for the required machine.
- (vi) The theoretical stalled torque of the machine, when supplied from the d.c. supply voltage calculated above, is then obtained by simple proportion using the nomogram Fig. 7, and the specified load torque required at the high speed limit of the working range.

The calculations in (v) and (vi) are explained in more detail in Appendix 3.

RELATIONSHIP BETWEEN MOTOR SPEED AND THYRISTOR FIRING ANGLE

The data presented in Fig. 4 have been replotted in the two other alternative forms in Fig. 8 and 9 where the relationship between thyristor firing angle and speed and torque is more clearly illustrated. The effects of different values of power factor cos φ are shown. A zero shaft torque line for $\alpha = 0{\cdot}05$ has also been added.

It may be seen from Fig. 8 that if true mechanical output torques are considered with respect to the shaft torque zero, the speed versus firing angle relationship becomes more linear even in the discontinuous conduction region. As would be expected from Fig. 4 the speed versus firing angle relationship becomes much more linear in the continuous conduction region, but it has already been shown that motor efficiency is very low at these high torques and low speeds. External series inductance can be used to extend this desirable region but the added expense of the inductor would be difficult to justify in most cases.

Many thyristor trigger circuits make use of an RC delay circuit to delay the thyristor firing pulse by the desired phase angle. The circuit will normally be designed so that it operates over the nearly linear part of the exponential charging curve so that firing angle will be a nearly linear function of resistance value. As it has been shown that motor speed is nearly a linear function of firing angle, it follows that a linear variable charging resistor can be calibrated in speed units and the scale will be nearly linear over the greater part of its range, provided that the working range of firing angle is suitably chosen.

Fig. 9 may be used to obtain, by a graphical method, the effect of speed stabilising feedback. Feedback systems are discussed below.

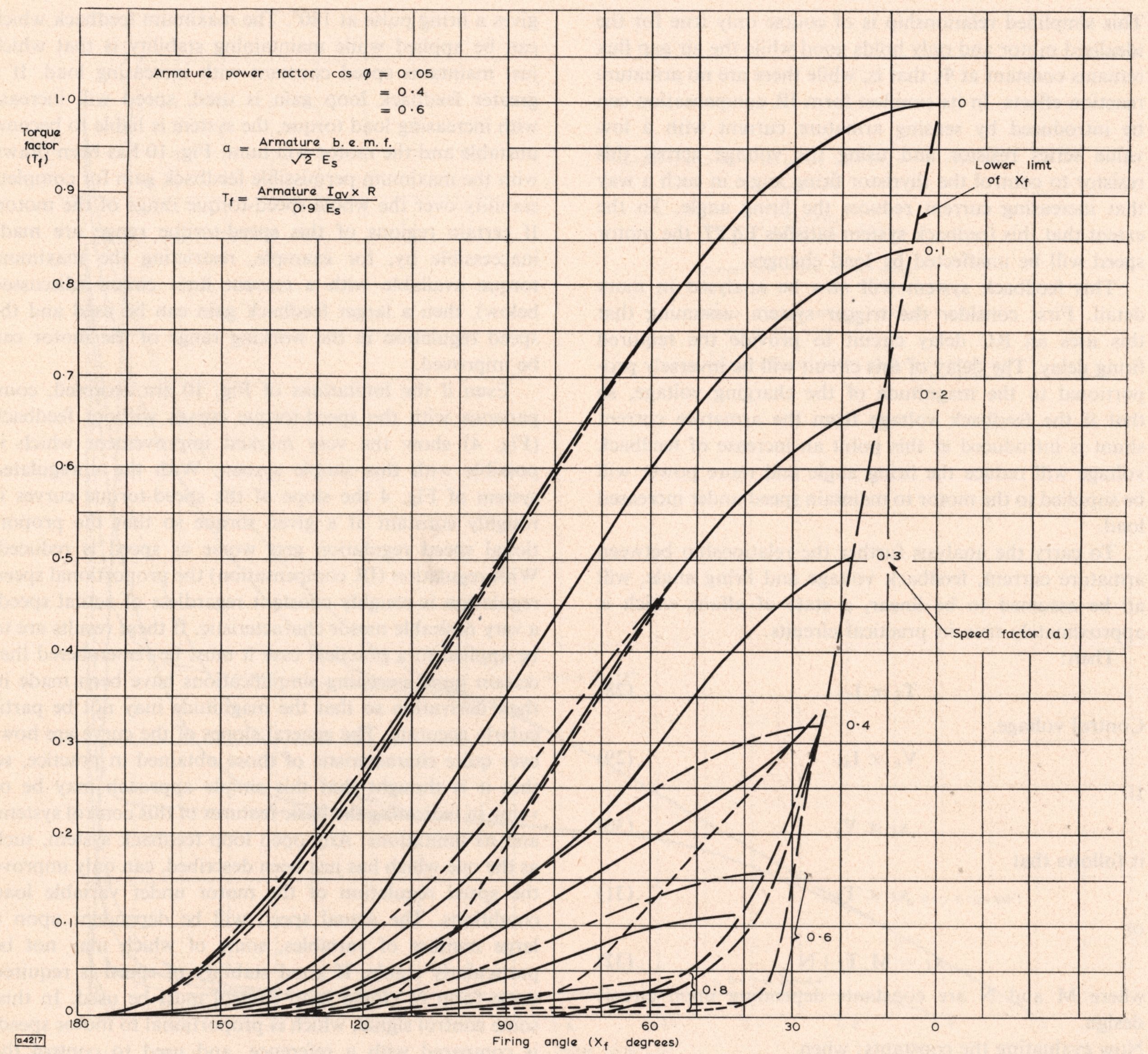


Fig. 9—Normalised torque versus firing angle curves for shunt motor, with speed factor as parameter

SUBSIDIARY CONTROL CIRCUITS

The control circuits considered are for stabilisation of speed and for current limiting. These are discussed separately.

Speed Stabilisation

Probably the simplest method for reducing the effects of load torque change upon speed is to use a form of IR compensation. The elementary motor equations, Eqs.1 and 2, show that the armature current, I, is directly proportional to torque T. The fall in speed with increasing torque is proportional to (E−IR). If therefore a way can be found to increase E at the same rate that IR increases so that

$$(E+\delta E)-(IR+\delta IR) = \text{Constant} \quad \ldots(27)$$

then the motor speed will be unaffected by load change.

This simplified relationship is of course only true for the idealised motor and only holds good while the air gap flux remains constant at Φ, that is, while there are no armature reaction effects. In its simplest form IR compensation can be introduced by sensing armature current with a low value series resistor and using the voltage across this resistor to control the thyristor firing angle in such a way that increasing current reduces the firing angle. To the extent that this feedback system satisfies Eq.27, the motor speed will be unaffected by load changes.

This feedback system will now be analysed in more detail. First consider the trigger system, assuming that this uses an RC delay circuit to provide the required firing delay. The delay of this circuit will be inversely proportional to the magnitude of the charging voltage, so that if the feedback voltage from the armature current shunt is introduced at this point an increase of feedback voltage will reduce the firing angle and more power will be supplied to the motor to maintain speed under increased load.

To carry the analysis further the relationship between armature current, feedback voltage and firing angle, will all be assumed to be linear, a state of affairs which is approximately true in practical circuits.

Then:

$$T_f \propto I_{av} \qquad \ldots(28)$$

Control voltage,

$$V_c \propto I_{av} \qquad \ldots(29)$$

If

$$x_f \propto V_c \qquad \ldots(30)$$

it follows that

$$x_f \propto T_F \qquad \ldots(31)$$

or

$$x_f = M.T_f + N \qquad \ldots(32)$$

where M and N are constants depending upon circuit design.

Now evaluating the constants; when

$$T_f = 0, \quad N = x_{f(o)}.$$

Therefore

$$M = \frac{x_{f(1)} - x_{f(o)}}{T_{f(1)}}. \qquad \ldots(33)$$

At $T_f = 0$, armature current is zero by definition, therefore there is no feedback voltage and $x_{f(o)}$ is determined only by the RC setting of the trigger circuit. In practical terms the constant N represents the trigger circuit firing angle at zero motor load, while constant M defines the feedback loop gain. In a variable speed drive constant N will normally be set by the speed control resistor and therefore change as speed is adjusted, while constant M will normally be fixed for all speeds. A family of feedback characteristics can be superimposed upon Fig. 9 where all lines have the same value of M (slope) but each has a different value of N. From the intersections of the feedback lines with the constant speed factor lines of Fig. 9 a new graph can be constructed (Fig. 10) relating speed and torque to feedback. Relative values of charging resistor R have also been included. The value R = 1 just gives a firing pulse at 180°. The maximum feedback which can be applied while maintaining stability is that which just maintains speed constant with increasing load. If a greater feedback loop gain is used, speed will increase with increasing load torque, the system is liable to become unstable and the motor will hunt. Fig. 10 has been drawn with the maximum permissible feedback gain for complete stability over the whole speed-torque range of the motor. If certain regions of this speed-torque range are made inaccessible by, for example, restricting the maximum torque available with a current limit circuit (discussed below), then a larger feedback gain can be used and the speed regulation in the working range of the motor can be improved.

Even if the limitations of Fig. 10 are accepted, comparisons with the speed-torque curves without feedback (Fig. 4) show the very marked improvement which is possible with this simple system. With the unregulated system of Fig. 4 the slope of the speed-torque curves is roughly constant at a given torque so that the proportional speed regulation gets worse as speed is reduced. With regulation (IR compensation) the proportional speed regulation is roughly constant regardless of actual speed, a very desirable motor characteristic. If these results are to be applied to a practical case it must be remembered that certain quite sweeping simplifications have been made in their derivation so that the magnitude may not be particularly accurate. The general slopes of the curve are however quite characteristic of those obtained in practice, so that it is thought that this simple approach may be of value in indicating the basic features of this control system, and its limitations. Any open loop feedback system, such as the one which has just been described, can only improve the speed regulation of the motor under variable load conditions. The actual speed will be dependent upon a large number of variables, some of which may not be particularly stable. If good stability of speed is required some form of closed loop control must be used. In this, some control signal, which is proportional to motor speed, is compared with a reference, and used to control the thyristor firing angle. Such systems are only affected by the motor characteristics in so far as the feedback loop gain required, for a particular degree of control, will depend upon the open loop regulation of the motor. The above characteristics, with or without IR drop compensation, can be used to obtain an estimate of the required loop gain. A knowledge of the required loop gain will enable an estimate to be made of the overall system stability. Further detailed discussion of the feedback control circuitry is outside the scope of this article which is intended to deal primarily with motor characteristics.

Current Limit

Various circuit arrangements can be designed which act in the reverse sense to the IR drop compensation arrangement. They are arranged so that the thyristor firing angle is increased as armature current increases, that is, as the motor load is increased less power is supplied to the motor. This type of control makes the motor speed regulation very poor. It is usually arranged to be effective only after a certain critical torque has been reached. The motor will stall at this torque with very little increase in supply current. This type of control circuit is generally used to protect the thyristor and rectifier control components, and the motor, against possible mechanical overloads and also to reduce the large currents, associated with starting, to a safe value. Other applications of current limit are to make inaccessible the higher torque region of the speed-torque range of the motor, so that a greater degree of IR drop compensation can be used while maintaining motor stability, and to provide unusual motor characteristics for certain drive requirements. One such requirement is found in certain winding machines or spool drivers where a constant tension has to be maintained while the spool diameter increases as the winding proceeds. By making the point at which current limit operates adjustable, a control of winding tension can be provided. An extension of this technique could be used to provide a predetermined rate of increase of tension for fine wire winding machines.

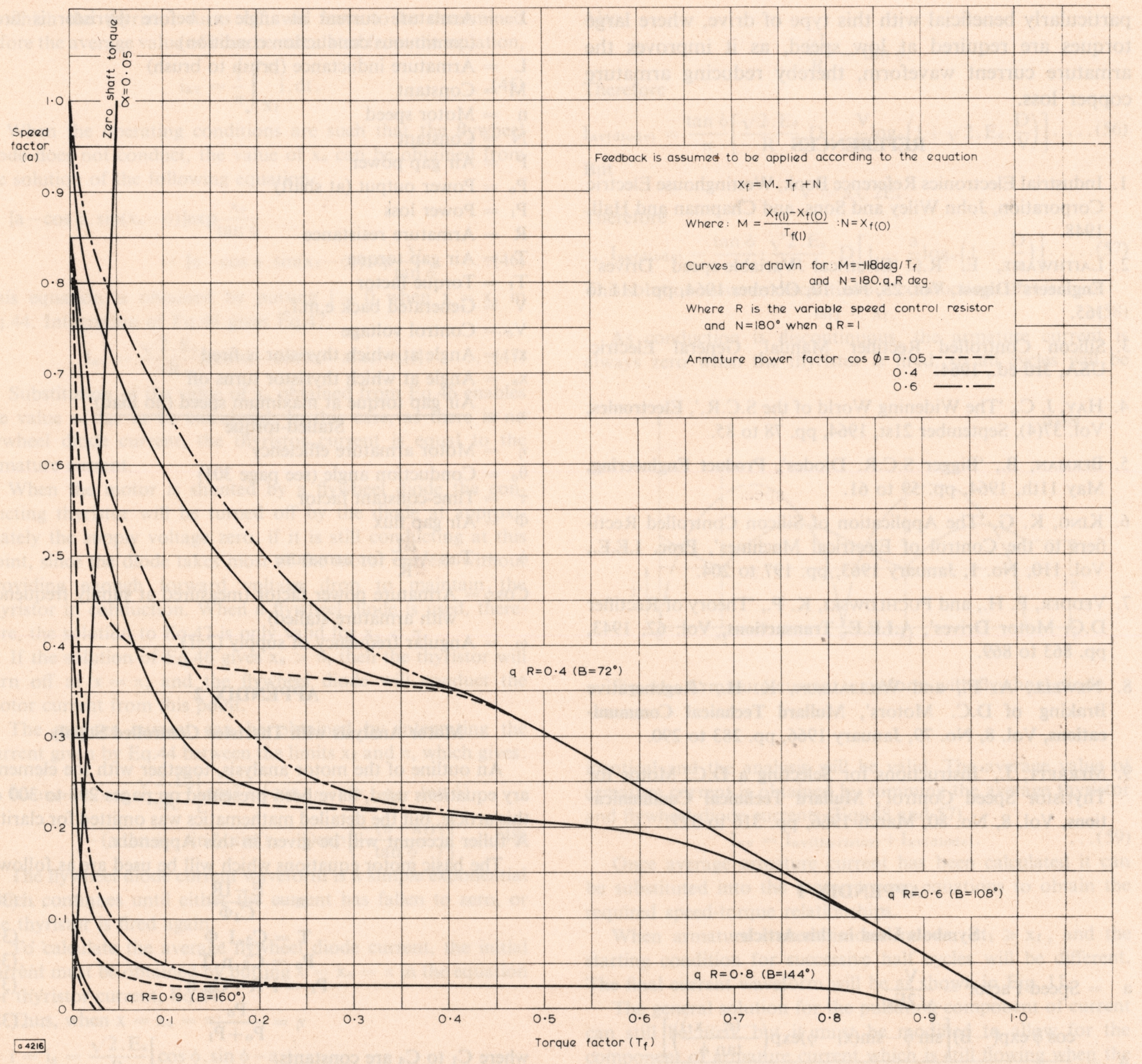


Fig. 10—Normalised speed versus torque for a thyristor controlled shunt motor with IR-drop compensation

This topic is discussed further in considerable detail in Reference 8.

Motor performance with current limit can be estimated with the aid of Fig. 10 as outlined for IR drop compensation, but inverting the phase of the feedback voltage so that the slope of the speed-torque characteristic is increased.

The most difficult systems to design are those which require a large speed range with maximum torque at the low speed end of the range, combined with good speed regulation at low speed. Speed ranges of 20:1 combined with a torque range of 1:10, that is torque at low speed ten times high speed torque, can be obtained with a motor efficiency greater than 40% over the speed range, even with ¼ hp machines. The use of a flywheel diode is particularly beneficial with this type of drive, where large torques are required at low speed, as it improves the armature current waveform, thereby reducing armature copper loss.

REFERENCES

1. Industrial Electronics Reference Book, Westinghouse Electric Corporation, John Wiley and Sons, and Chapman and Hall, 1948.
2. LAITHWAITE, E. R., 'Electrical Variable-speed Drives', **Engineers' Digest,** Vol. 25, No. 10, October 1964, pp. 111 to 165.
3. Silicon Controlled Rectifier Manual, General Electric USA, 3rd ed., 1964.
4. HAY, J. C., 'The Widening World of the S.C.R.', **Electronics,** Vol. 37(4), September 21st, 1964, pp. 78 to 85.
5. BERMAN, B., 'Bigger S.C.R. Diodes', **Product Engineering,** May 11th, 1964, pp. 59 to 61.
6. KING, K. G. 'The Application of Silicon Controlled Rectifiers to the Control of Electrical Machines', **Proc. I.E.E.,** Vol. 110, No. 1, January 1963, pp. 197 to 204.
7. VEDDER, E. H., and PUCHLOWSKI, K. P., 'Theory of Rectifier D.C. Motor Drives', **A.I.E.E. Transactions,** Vol. 62, 1943, pp. 863 to 869.
8. NEWELL, A. F., and WILLIAMSON, K. H., 'Regenerative Braking of D.C. Motors', **Mullard Technical Communications,** Vol. 8, No. 79, January 1966, pp. 283 to 290.
9. MERRETT, J., 'Instructions for Selecting a D.C. Motor for Thyristor Speed Control', **Mullard Technical Communications,** Vol. 8, No. 80, March 1966, pp. 316 to 327.

APPENDIX 1

Symbols Used in this Article

a = Speed Factor = $\dfrac{V}{\sqrt{2}E_s}$

$$A = \frac{\cos\phi\exp(-B)\left[\sin\phi - \sin(x_f-\phi)\exp\left(\dfrac{-(\pi-x_f)}{\tan\phi}\right)\right]}{\left[1-\exp(-B).\exp\left(\dfrac{-(\pi-x_f)}{\tan\phi}\right)\right]}$$

$$B = \frac{x_f.R}{\omega L} = \frac{x_f}{\tan\phi}$$

$\left.\begin{matrix} C_1 \\ C_2 \\ C_3 \\ C_4 \end{matrix}\right\}$ = Constants

$$D = \left\{\cos\phi.\sin\phi - a + [a-\cos\phi.\sin(x_f-\phi)]\exp\left(\frac{-(\pi-x_f)}{\tan\phi}\right)\right\}$$

e = Instantaneous value of supply voltage at angle x, such that $e = E\sin x$
E = Supply voltage d.c.
E_s = Supply voltage (r.m.s.)
f = Supply frequency
F_f = Armature current form factor
i = Instantaneous supply current at any angle x
i_s = Instantaneous supply current at angle x_s
i_s' = Instantaneous armature current at angle x_s (continuous conduction condition)
I = Armature current d.c.
I_{av} = Average armature current
I_f = Armature current factor
I' = Armature current at angle x_f before thyristor is fired (continuous conduction condition)
L = Armature inductance (brush to brush)
M = Constant
n = Motor speed
N = Constant
P = Air gap power
P_o = Power output (at shaft)
P_l = Power loss
R = Armature resistance
T = Air gap torque
T_f = Torque factor
V = Generated back e.m.f.
V_c = Control voltage
x_f = Angle at which thyristor is fired
x_s = Angle at which thyristor turns off
α = $\dfrac{\text{Air gap torque at maximum speed (no load)}}{\text{Stalled-torque}}$
η = Motor armature efficiency
θ_c = Conduction angle (see page 301)
τ = Time-constant factor
Φ = Air gap flux
ϕ = $\text{Tan}^{-1}\dfrac{\omega L}{R}$ for armature
$\text{Cos}\phi$ = Armature power factor (measured at supply frequency with armature stalled)
ω = Angular frequency of supply ($2\pi f$)

APPENDIX 2

Motor Analysis with Thyristor Controlled Supply

An outline of the motor analysis, together with the elementary equations used, have been presented on pages 298 to 300 of this article, but the detailed mathematics was omitted for clarity. A fuller account will be given in this Appendix.

The basic motor equations which will be used are as follows,

$$n = \frac{E-IR}{C_1\Phi} \qquad \ldots(34)$$

$$T = C_2.I.\Phi \qquad \ldots(35)$$

$$P_o = C_3.n.T \qquad \ldots(36)$$

$$P_l = I^2R + C_4.n \qquad \ldots(37)$$

$$\eta = \frac{P_o}{P_o+P_l} \qquad \ldots(38)$$

where C_1 to C_4 are constants.

Other terms used are defined as follows;

$$\text{Power factor } \cos\phi \text{ is given by } \phi = \tan^{-1}\frac{\omega L}{R} \qquad \ldots(39)$$

$$\text{Speed factor } a = \frac{V}{\sqrt{2}.E_s} \qquad \ldots(40)$$

$$\text{Torque factor } T_f = \frac{I_{av}.R}{0{\cdot}9\,E_s} \qquad \ldots(41)$$

$$\text{Form factor } F_f = \frac{I_{r.m.s.}}{I_{av}}. \qquad \ldots(42)$$

Also, in general, for an a.c. supply

$$e = \sqrt{2}.E_s.\sin x. \qquad \ldots(43)$$

When a reactive load is supplied from an a.c. voltage source in such a way that the supply is connected to the load repetitively at some intermediate point in the cycle (angle x_f) and the connection to the supply is broken at some other point (angle x_s), not necessarily at the voltage zero, the instantaneous supply current can be represented by the equation below.

$$i = \frac{\sqrt{\ }.E_s}{R}\left\{\cos\phi.\sin(x-\phi) - a + \right.$$
$$\left. + [a-\cos\phi.\sin(x_f-\phi)]\exp\frac{-(x-x_f)}{\tan\phi}\right\}. \qquad \ldots(44)$$

The value of x_f will be known, as it is determined in practice by the thyristor trigger circuit, but the limiting value x_s of x, at which the thyristor turns off, will have to be determined before the average value of i can be calculated from the equation,

$$i_{av} = \frac{1}{\pi}\int_{x_f}^{x_s} i.dt. \qquad \ldots(45)$$

When the operating conditions are such that the flywheel diode does not conduct, the value of x_s can be obtained from the solution of the following equation:

$$[a-\cos\phi.\sin(x_s-\phi)]\exp\frac{x_s}{\tan\phi}$$
$$= [a-\cos\phi.\sin(x_f-\phi)]\exp\frac{x_f}{\tan\phi}. \qquad \ldots(46)$$

This equation is obtained by putting $i = 0$ when $x = x_s$ in Eq.44. Integration of Eq.44 gives Eq.47.

$$I_{av} = \frac{\sqrt{2}.E_s}{\pi R}[\cos x_f - \cos x_s - a(x_s-x_f)]. \qquad \ldots(47)$$

Substitution of the value of x_s obtained from Eq.46 enables the value of I_{av} to be calculated. In this case, as there is no flywheel diode current, the thyristor current is equal to the armature current.

When the motor is shunted by a flywheel diode the conducting thyristor will be turned off by the diode at approximately the supply voltage zero, if it is still conducting at this point, since the diode takes most of the motor current without providing enough forward voltage drop to maintain the thyristor in conduction. When a flywheel diode is used, therefore, the solution to Eq.47 is only valid for $x_s < \pi$.

If the solution of Eq.46 gives $x_s > \pi$, then the thyristor will turn off at $x = \pi$, and the flywheel diode will conduct the motor current from this point.

The average current can be obtained by integrating the current given by Eq.44 between the limits x_f and π, which gives:

$$I_{av(thyristor)} = \frac{\sqrt{2}.E_s}{\pi R}\left[\cos^2 + \phi\cos x_f - a(\pi - x_f - \tan\phi) - \right.$$
$$\left. -(a\tan\phi - \sin\phi.\sin(x_f-\phi)\exp\left(\frac{-(\pi-x_f)}{\tan\phi}\right)\right]. \qquad \ldots(48)$$

The flywheel diode current waveform is a simple exponential which continues until either the current has fallen to zero, or the thyristor is fired again.

To calculate the average flywheel diode current, the initial current must be obtained by putting $x = x_s = \pi$ in the equation for thyristor current, Eq.44.

Thus, when $x = x_s = \pi$:

$$i = i_s = \frac{\sqrt{2}.E_s}{R}\left[\cos\phi.\sin\phi - a + \right.$$
$$\left. + [a-\cos\phi.\sin(x_f-\phi)]\exp\left(\frac{-(\pi-x_f)}{\tan\phi}\right)\right]. \qquad \ldots(49)$$

The diode current falls exponentially from this value at a rate determined by motor back e.m.f. (V) and the armature resistance (R) such that:

$$i_{diode} = (i_s + V/R)\exp(-tR/L). \qquad \ldots(50)$$

The diode becomes reverse-biased and stops conducting when,

$$i_{diode} = V/R. \qquad \ldots(51)$$

The maximum conduction time of the diode is therefore

$$t_1 = L/R\,\log_e(1 + i_s.R/V). \qquad \ldots(52)$$

Integration of Eq.50 and substitution for t from Eq.52 gives,

$$I_{av(diode)} = \frac{(\omega L/R)\log_e(1+i_s.R/V)}{\pi}\left[\frac{i_s}{\log_e(1+i_s.R/V)} - V/R\right]. \qquad \ldots(53)$$

This equation may be simplified in the following way. From Eq.49

$$i_s = \frac{\sqrt{2}.E_s}{R}.D \qquad \ldots(54)$$

where

$$D = \left[\cos\phi.\sin\phi - a + \right.$$
$$\left. + [a-\cos\phi.\sin(x_f-\phi)]\exp\left(\frac{-(\pi-x_f)}{\tan\phi}\right)\right]. \qquad \ldots(55)$$

Also

$$\frac{\omega L}{R} = \tan\phi.$$

Therefore

$$I_{av(diode)} = \frac{\tan\phi}{\pi}\left[\frac{\sqrt{2}.E_s}{R}.D - \frac{V}{R}\log_e\left(1+\sqrt{2}.E_s.\frac{D}{V}\right)\right]. \qquad \ldots(56)$$

But

$$V = a\sqrt{2}E_s.$$

Therefore

$$I_{av(diode)} = \frac{\tan\phi}{\pi}\cdot\frac{\sqrt{2}.E_s}{R}\cdot D\left[1 - \frac{a}{D}\log_e\left(1+\frac{D}{a}\right)\right]. \qquad \ldots(57)$$

This solution assumes discontinuous conduction, that is:

$$\omega t_1 < x_f. \qquad \ldots(58)$$

As conduction is discontinuous, the armature current is always zero when the thyristor is fired, so all cycles will be identical and the analysis will be valid. The average value of armature current is obtained by summing the average thyristor and flywheel diode currents (Eqs.48 and 57).

$$I_{av} = I_{av(thyristor)} + I_{av(diode)}. \qquad \ldots(59)$$

Once average armature current has been calculated it can be substituted into the motor torque equations to obtain the required speed-torque relationships.

When armature current is continuous, $\omega t_1 \geqslant x_f$, and the starting condition for successive half cycles will be different. The final current waveform will be as shown in Fig. 11.

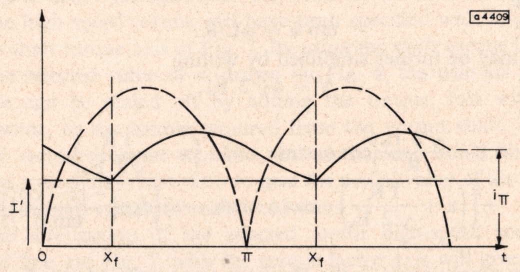


Fig. 11—Armature current waveform with flywheel diode

The general solution for the pulsating component of current can still be used, but it must be modified to allow for the component of armature current which is still flowing when the thyristor is fired again. This current is shown in Fig. 11 with magnitude I′.

The basic differential equation from which the thyristor current equation was derived is:

$$\sqrt{2}.E_s.\sin x - V = L\frac{di}{dt} + iR. \qquad \ldots(60)$$

The solution of this equation gives:

$$i = \frac{\sqrt{2}.E_s}{R}\left\{\cos\phi.\sin(x-\phi) - \frac{V}{\sqrt{2}.E_s} + \right.$$
$$\left. + \left[\frac{V}{\sqrt{2}.E_s} - \cos\phi.\sin(x_f-\phi)\right]\exp\left(-\frac{R}{L}.t\right)\right\}. \qquad \ldots(61)$$

If the current at angle $x_f = I'$, Eq.60 can be rewritten in the following way:

$$\sqrt{2}.E_s.\sin x - V = L\frac{di}{dt} + (i+I')R \qquad \ldots(62)$$

or

$$\sqrt{2}.E_s.\sin x - (V+I'R) + L\frac{di}{dt} + iR. \qquad \ldots(63)$$

The solution of this equation gives

$$i = I' + \frac{\sqrt{2}.E_s}{R}\left\{\cos\phi.\sin(x-\phi) - \frac{V+I'.R}{\sqrt{2}.E_s} + \right.$$
$$\left. + \left[\frac{V+I'.R}{\sqrt{2}.E_s} - \cos\phi.\sin(x_f-\phi)\right]\exp\left(-\frac{R}{L}.t\right)\right\}. \qquad \ldots(64)$$

This solution is not exact as it assumes that the initial current (I) was a steady current, that is, with zero rate of change. In practice there would be a small negative di/dt component which should be taken into account, but the error introduced by neglecting this component is small and the simplification in the analysis is considerable.

The next step in the solution is to find an expression for the current I′.

Let the thyristor instantaneous current at angle π be given by:

$$i = i_\pi.$$

Then:

$$i_\pi = I' + \frac{\sqrt{2}.E_s}{R}\left\{\cos\phi.\sin(\pi-\phi) - \frac{V+I'.R}{\sqrt{2}.E_s} + \right.$$
$$\left. + \left[\frac{V+I'.R}{\sqrt{2}.E_s} - \cos\phi.\sin(x_f-\phi)\right]\exp\left(-\frac{(\pi-x_f)}{\tan\phi}\right)\right\} \quad \ldots(65)$$

where

$$\tan\phi = \omega L/R.$$

This may be further simplified by writing

$$V = a\sqrt{2}.E_s.$$

Then

$$i_\pi = I' + \frac{\sqrt{2}.E_s}{R}\left\{\cos\phi.\sin(\pi-\phi) - \left(a + \frac{I'R}{\sqrt{2}.E_s}\right) + \right.$$
$$\left. + \left[\left(a + \frac{I'R}{\sqrt{2}.E_s}\right) - \cos\phi.\sin(x_f-\phi)\right]\exp\left(\frac{-(\pi-x_f)}{\tan\phi}\right)\right\}. \quad \ldots(66)$$

At supply angle π the thyristor turns off and the flywheel diode starts to conduct. The load current from this point decays exponentially according to the relationship:

$$I' = \left[\left(i_\pi + \frac{V}{R}\right)\exp\left(-t\frac{R}{L}\right)\right] - \frac{V}{R} \quad \ldots(67)$$

where $t = x_f/\omega$ and I' = diode current at angle x_f or

$$I' = \left[\left(i_\pi + \frac{a\sqrt{2}.E_s}{R}\right)\exp\left(\frac{-x_f}{\tan\phi}\right)\right] - \frac{a\sqrt{2}.E_s}{R}. \quad \ldots(68)$$

Put

$$\frac{x_f}{\tan\phi} = B$$

and rearrange. Then,

$$I' = \left\{i_\pi \exp(-B) - \frac{a\sqrt{2}.E_s}{R}[1-\exp(-B)]\right\}. \quad \ldots(69)$$

Substitution in Eq.66 gives the following:

$$i_\pi = \left\{\frac{\frac{\sqrt{2}.E_s}{R}.\cos\phi\left[\sin(\pi-\phi) - \sin(x_f-\phi).\exp\left(\frac{-(\pi-x_f)}{\tan\phi}\right)\right]}{\left[1-\exp(-B).\exp\left(\frac{-(\pi-x_f)}{\tan\phi}\right)\right]}\right\} -$$
$$- \frac{a\sqrt{2}.E_s}{R}. \quad \ldots(70)$$

Substituting this value of i_π into Eq.69 and simplifying gives,

$$I' = \frac{\sqrt{2}.E_s}{R}\left\{\frac{\cos\phi.\exp(-B)}{\{1-\exp(-B).\exp[-(\pi-x_f)]\}}.\right.$$
$$\left.\left[\sin\phi - \sin(x_f-\phi)\exp\left(\frac{-(\pi-x_f)}{\tan\phi}\right)\right] - a\right\}. \quad \ldots(71)$$

This value of I′ may now be used to evaluate the instantaneous thyristor current i given by Eq.64.

The average thyristor current is given by:

$$I_{av(thyristor)} = \frac{1}{\pi}\int_{x_f}^{\pi} i.dt. \quad \ldots(72)$$

Integration and insertion of limits gives the following,

$$I_{av(thyristor)} = \left(\frac{\pi-x_f}{\pi}\right)I' + \frac{\sqrt{2}.E_s}{\pi R}\left\{\cos^2\phi + \cos\ x_f - \right.$$
$$-\left(a + \frac{I'R}{\sqrt{2}.E_s}\right)\times(\pi - x_f - \tan\ \phi) -$$
$$\left. -\left[\left(a + \frac{I'.R}{2E_s}\right)\tan\phi - \sin\phi.\sin(x_f-\phi)\right]\exp\left(\frac{-(\pi-x_f)}{\tan\phi}\right)\right\} \quad \ldots(73)$$

where I′ is given by Eq.71.

To simplify this expression, put

$$A = \frac{\cos\phi.\exp(-B)\left[\sin\phi - \sin(x_f-\phi)\exp\left(\frac{-(\pi-x_f)}{\tan\phi}\right)\right]}{\left[1-\exp(-B).\exp\left(\frac{-(\pi-x_f)}{\tan\phi}\right)\right]}. \quad \ldots(74)$$

Then

$$i_\pi = \frac{\sqrt{2}.E_s}{R}\left[\frac{A}{\exp(-B)} - a\right] \quad \ldots(75)$$

$$I' = \frac{\sqrt{2}.E_s}{R}\left(A-a\right). \quad \ldots(76)$$

Substitution into Eq.73 gives,

$$I_{av(thyristor)} = \frac{\sqrt{2}.E_s}{\pi R}\left\{(\pi-x_f)(A-a) + \cos^2\phi + \cos x_f - \right.$$
$$\left. - A(\pi - x_f - \tan\phi) - [A.\tan\phi - \sin\varphi.\sin(x_f-\phi)]\exp\left(\frac{-(\pi-x_f)}{\tan\phi}\right)\right\}. \quad \ldots(77)$$

The flywheel diode average current is given approximately by the equation,

$$I_{av(diode)} = \frac{x_f}{\pi}\left\{\frac{i_\pi + I'}{2}\right\}. \quad \ldots(78)$$

Therefore

$$I_{av(diode)} = \frac{\sqrt{2}.E_s}{\pi R}.\frac{x_f}{2}\left\{A\left(1 + \frac{1}{\exp(-B)}\right) - 2a\right\}. \quad \ldots(79)$$

The average armature current can be obtained as before by summing the average thyristor and diode currents given by Eqs.77 and 79.

$$I_{av} = I_{av(thyristor)} + I_{av(diode)}. \quad \ldots(80)$$

In all cases it must be remembered that the thyristor turn-on angle x_f is restricted by the motor generated back e.m.f., V. The thyristor can only be fired when the instantaneous value of the supply voltage is greater than the motor back e.m.f., V.

Thus the following relationship must always be satisfied:

$$\sin^{-1}a < x_f < \pi - \sin^{-1}a. \quad \ldots(81)$$

The equations for the average currents of the armature, thyristor and flywheel diode are listed below for more convenient reference.

Conduction Type I, where $x_s < \pi$.

Flywheel diode does not conduct.

$$I_{av(thyristor)} = \frac{\sqrt{2}\ E_s}{\pi R}\left[\cos x_f - \cos x_s - a(x_s - x_f)\right] \quad \ldots(82)$$

$$I_{av(diode)} = 0 \quad \ldots(83)$$

$$I_{av} = I_{av(thyristor)}. \quad \ldots(84)$$

Conduction Type II, where $x_s > \pi$.

Flywheel diode conducts; armature current is discontinuous.

$$I_{av(thyristor)} = \frac{\sqrt{2}.E_s}{\pi R}\left[\cos^2\phi + \cos x_f - a(\pi - x_f - \tan\phi) - \right.$$
$$\left. - [a\tan\phi - \sin\phi.\sin(x_f - \phi)]\exp\left(\frac{-(\pi-x_f)}{\tan\phi}\right)\right] \quad \ldots(85)$$

$$I_{av(diode)} = \frac{\sqrt{2}.E_s}{\pi R}.D.\tan\phi\left[1 - \frac{a}{D}\log_e\left(1 + \frac{D}{a}\right)\right] \quad \ldots(86)$$

$$I_{av} = I_{av(thyristor)} + I_{av(diode)} \quad \ldots(87)$$

Conduction Type III, where $x_s > \pi$.

Flywheel diode conducts; armature current is continuous.

$$I_{av(thyristor)} = \frac{\sqrt{2}.E_s}{\pi R}\left\{(\pi - x_f)(A-a) + \cos^2\phi + \cos x_f - \right.$$
$$- A(\pi - x_f - \tan\phi) -$$
$$\left. - [A\tan\phi - \sin\phi.\sin(x_f-\phi)]\exp\left(\frac{-(\pi-x_f)}{\tan\phi}\right)\right\} \quad \ldots(88)$$

$$I_{av(diode} = \frac{\sqrt{2}.E_s}{\pi R}.\frac{x_f}{2}\left[A\left(1 + \frac{1}{\exp(-B)}\right) - 2a\right] \quad \ldots(89)$$

$$I_{av} = I_{av(thyristor)} + I_{av(diode)}, \quad \ldots(90)$$

where

$$A = \frac{\cos\phi.\exp(-B)\left[\sin\phi - \sin(x_f-\phi)\exp\left(\frac{-(\pi-x_f)}{\tan\phi}\right)\right]}{\left[1-\exp(-B).\exp\left(\frac{-(\pi-x_f)}{\tan\phi}\right)\right]} \quad \ldots(91)$$

$$B = \frac{x_f}{\tan\phi} \quad \ldots(92)$$

$$D = \left\{\cos\phi.\sin\phi - a + [a - \cos\phi.\sin(x_f-\phi)]\exp\left(\frac{-(\pi-x_f)}{\tan\phi}\right)\right\}. \quad \ldots(93)$$

These equations for average armature, thyristor and flywheel diode current have been solved by computer for a wide range of values of firing angle x_f and speed factor a at a number of different armature power factors $\cos\phi$. Some of the results have been presented graphically in Figs. 4 and 5. In Fig. 4 the results have been plotted in terms of a torque factor, T_f, where $T_f = 1$ corresponds with maximum average armature current. The following relationships therefore apply

$$I_{av(max)} = \frac{2}{\pi}.\frac{\sqrt{2E_s}}{R}$$

$$\text{Torque factor} = I_{av}\Big/\frac{2}{\pi}.\frac{\sqrt{2E_s}}{R}$$
$$= I_{av}.\frac{\pi}{2}.\frac{R}{\sqrt{2E_s}}.$$

APPENDIX 3

The Estimation of Equivalent D.C. Supply Voltage for a Motor Supplied from Phase Controlled A.C. Mains

A procedure for arriving at a suitable working range for a thyristor controlled motor has been described in detail on page 304. The final step of arriving at the specification of the d.c. machine which will satisfy these requirements has not, however, been dealt with. The use of Figs. 4 and 7, in reverse, to obtain the motor voltage will be described in more detail in this Appendix.

It will be remembered that in Fig. 4 the zero torque axis corresponds with zero air gap torque (zero armature current), while in Fig. 7 the zero torque axis represents zero available shaft torque. The speed-efficiency lines on Fig. 7 have been defined in terms of a factor 'α' where:

$$\alpha = \frac{\text{Air gap torque at maximum speed at no-load}}{\text{Stalled torque}}.$$

The power loss at no-load is made up from friction, windage, iron- and copper-losses which, for simplicity, have all been combined and assumed to be linearly proportional to speed. In practice this assumption is approximately true provided that the resulting value of α is not large and that the motor is not a small high-speed machine. This simple approximation does not take into account friction losses at low speed, but the error is not great. By making use of the factor α, zero mechanical output torque lines can be drawn on Fig. 4; one is shown for $\alpha = 0{\cdot}05$. Now if d.c. motor speed-torque characteristics are superimposed upon Fig. 4 as straight parallel diagonal lines they will intersect the shaft torque and air gap torque axes at two different values of speed factor. Speed factor is however proportional to d.c. applied voltage. Therefore the intersection with the shaft torque axis will correspond with the true no-load speed while the intersection with the zero air gap torque axis will correspond with the required d.c. supply voltage. The difference between these two intersections, expressed in terms of voltage, will be the difference between the applied d.c. voltage and the motor back e.m.f. This graphical method then allows us to obtain the supply voltage required to drive a motor at a certain speed when the value of α is known.

The speed-torque curves of Fig. 4 have been drawn so that a speed factor, a = 1, corresponds with a voltage equal to the peak value of the a.c. supply. When transferring speed factor values from Fig. 7 to Fig. 4 they must be multiplied by $2/\pi$.

When the operating speed and the corresponding torque have been fixed in terms of speed and torque factors on Fig. 7, a speed-torque line which passes through this operating point can be redrawn on Fig. 4. This line intersects the zero shaft torque line, with the appropriate α value, at the speed factor value corresponding with zero torque on Fig. 7. The intersection of this speed-torque line, on Fig. 4, with the air gap torque axis then gives the required d.c. supply voltage as a fraction of the a.c. peak supply voltage.

Having obtained the value of d.c. supply voltage for which the motor should be designed to give the highest speed required, it now remains to obtain the value of stalled torque at this voltage to define the equivalent d.c. shunt motor characteristics.

The high speed torque will have been specified with respect to the shaft torque axis of Fig. 7. By using the shaft torque line, for the selected value of α, drawn on Fig. 4, the true air gap torque can be scaled off by adding the torque, lost within the motor, to the torque required from the output shaft. The actual output torque required, in mechanical units (lb.ft), is then scaled up in this ratio to give the actual value of air gap torque at high speed in mechanical units.

The intersection of the selected motor high-speed speed-torque line, on Fig. 7, with the torque-factor axis will give the stalled-torque torque-factor. The air gap torque at high speed can now be scaled to give the true air gap stalled-torque of the motor in terms of mechanical units.

We have now specified the d.c. supply voltage, the stalled torque, and the motor efficiency at full load for the simplified machine model. To the extent that this model represents an actual motor, the required motor has now been completely specified in terms of d.c. machine data. (It should be remembered, however, that the calculated efficiency neglects the power required to energise the field.) In practice a machine designer will know the extent to which a particular machine departs from the idealised model, so that he will be able to make the necessary corrections to enable the motor, when built, to have a characteristic which approximates quite closely to the idealised characteristic in those working regions similar to d.c. machine operation.

When the motor is driven from an a.c. supply with thyristor control this design procedure will only be approximate, but should provide a useful working guide. Further work has been planned to measure the characteristics of a number of motors of different speeds and ratings to test the validity of this design approach and to obtain any necessary empirical correction factors. Practical experience to date with a small number of fractional h.p. machines indicates that quite good agreement is obtained between theory and practice.

APPENDIX 4

Miscellaneous Data

The following relationships between power, torque, current and voltage are of assistance when selecting the motor design.

$$\begin{aligned} 1\ \text{hp} &= 33\,000\ \text{ft.lb/min} \\ &= 550\ \text{ft.lb/s} \\ &= 4\,550\ \text{kg m/min} \\ &= 76\ \text{kg m/s} \\ &= 746\ \text{W} \end{aligned}$$

$$\text{Torque} = \frac{\text{Horse Power} \times 33\,000}{2\pi \times \text{Speed (rev/min)}}\ \text{lb.ft.}$$
$$= \frac{5250 \times \text{H.P.}}{\text{Speed}}\ \text{lb.ft.}$$

Current:

A useful working rule is 5 amps per hp

For a 220V motor this implies an armature efficiency of 68%.

Combining the Torque and Current equations gives approximately

$$\text{Current/unit torque} = \frac{\text{Speed (rev/min)}}{1000}\ \text{A/lb.ft.}$$

It is important to remember that these relationships between current and torque only apply for a motor running at its full rated speed. For a thyristor controlled motor running at reduced speed the armature current may be many times higher.

The following data are required to fully provide the specifications for a d.c. shunt motor.

(i) d.c. supply voltage
(ii) Full load torque
(iii) Full load speed
(iv) Stalled torque
(v) No load speed

Basic Data

(vi) Full load efficiency
(vii) Continuous or intermittent rating
(viii) If intermittent, number of starts per hour
(ix) Reliability
(x) Environment
(xi) Any special requirements, such as reversing

Secondary Characteristics

In addition to the above list the following will be required to fully specify a shunt motor for thyristor speed control.

(i) A.C. supply voltage
(ii) Limits of working speed range
(iii) Upper limits of torque, at the upper and lower limits of the speed range
(iv) Speed regulation required at the upper and lower limits of the speed range. If the working torque range is normally restricted to less than the full range from zero to maximum, the lower torque limit should be stated as this may considerably simplify the design.
(v) The duty cycle, at different speeds if relevant.

APPENDIX 5

Calculation of Thyristor, Flywheel Diode and Armature R.M.S. Current and Armature Current Form Factor

Thyristor r.m.s. Current

The general equation for instantaneous thyristor current is

$$i = \frac{\sqrt{2}.E_s}{R}\left\{[\cos\phi.\sin(x-\phi)-a]+ +[a-\cos\phi.\sin(x_f-\phi)]\exp\left(\frac{-(x-x_f)}{\tan\phi}\right)\right\} \quad \ldots(94)$$

To obtain the r.m.s. current, the square of the instantaneous current must be integrated between the appropriate limits to obtain the mean squared value.

To simplify the working, put

$$\frac{x-x_f}{\tan\phi} = X \quad \ldots(95)$$

and

$$[a-\cos\phi.\sin(x_f-\phi)] = Q \quad \ldots(96)$$

Then,

$$i = \frac{\sqrt{2}.E_s}{R}\{[\cos\phi.\sin(x-\phi)-a]+Q.\exp(-X)\} \quad \ldots(97)$$

Therefore,

$$i^2 = \frac{2.E_s^2}{R}\{[\cos\phi.\sin(x-\phi)-a]^2+Q^2.\exp(-2X)+ +2[\cos\phi.\sin(x-\phi)-a]Q.\exp(-X)\}. \quad \ldots(98)$$

Expansion, and collection of terms, gives:

$$i^2 = \frac{2.E_s^2}{R^2}[(a^2+\tfrac{1}{2}\cos^2\phi)-\tfrac{1}{2}\cos^2\phi.\cos^2(x-\phi)-2a\cos\phi. .\sin(x-\phi)+\exp(-2X).Q^2-2a\exp(-X).Q+2\cos\phi. .\sin(x-\phi).\exp(-X).Q] \quad \ldots(99)$$

$$I^2_{r.m.s.} = \frac{1}{\pi}\int_{x=x_f}^{x=x_s} i^2.dx. \quad \ldots(100)$$

All the terms of the integration are simple integrals, with the exception of the last, for which the following standard integral may be used to obtain a solution.

$$\exp(\alpha x).\sin\beta x.dx = \exp(\alpha x)\left(\frac{\alpha\sin\beta x-\beta\cos\beta x}{\alpha^2+\beta^2}\right) \quad \ldots(101)$$

Integrating and simplifying:

$$I^2_{r.m.s.} = \frac{2.E_s^2}{\pi R^2}\Big[(a^2+\tfrac{1}{2}\cos^2\phi)x+\cos\phi[2a\cos(x-\phi)- -\tfrac{1}{4}\cos\phi.\sin 2(x-\phi)] - \frac{\tan\phi}{2}.\exp(-2X).Q^2+ +2a\tan\phi.\exp(-X).Q-2\exp(-X).\sin\phi. .\cos\phi.\sin x.Q\Big]_{x=x_f}^{x=x_s} \quad \ldots(102)$$

Type I Conduction (see page 299), insert limits $x = x_f$, $x = x_s$

$$I^2_{r.m.s.(thyristor)} = \frac{2.E_s^2}{\pi R^2}\Big\{(a^2+\tfrac{1}{2}\cos^2\phi)(x_s-x_f)+ +\cos\phi[2a.\cos(x_s-\phi)-\tfrac{1}{4}\cos\phi.\sin 2(x_s-\phi)]- -\cos\phi[2a.\cos(x_f-\phi)-\tfrac{1}{4}\cos\phi.\sin 2(x_f-\phi)]+ +\frac{\tan\phi}{2}\left[1-\exp\left(\frac{-2(x_f-x_f)}{\tan\phi}\right)\right].Q^2- -2a\tan\phi\left[1-\exp\left(\frac{-(x_s-x_f)}{\tan\phi}\right)\right].Q+2\sin\phi.\cos\phi. .\left[\sin x_f-\exp\left(\frac{-(x_x-x_f)}{\tan\phi}\right).\sin x_s\right].Q\Big\}. \quad \ldots(103)$$

Type II Conduction (see page 299), limits $x = x_f$, $x = \pi$

$$I^2_{r.m.s.(thyristor)} = \frac{2.E_s^2}{\pi R^2}\Big\{(a^2+\tfrac{1}{2}\cos^2\phi)(\pi-x_f)- -\cos^2\phi[2a-\tfrac{1}{4}\sin^2\phi]-\cos\phi[2a\cos(x_f-\phi)- -\tfrac{1}{4}\cos\phi.\sin 2(x_f-\phi)] + \frac{\tan\phi}{2}.\left[1-\exp\left(\frac{-2(\pi-x_f)}{\tan\phi}\right)\right]. .Q^2.-2a\tan\phi\left[1-\exp\left(\frac{-(\pi-x_f)}{\tan\phi}\right)\right].Q+ +2\sin\phi.\cos\phi.\sin x_f.Q\Big\}. \quad \ldots(104)$$

In this case the flywheel diode will conduct and the flywheel diode r.m.s. current must be evaluated before the motor armature current can be calculated.

For the flywheel diode:

$$i = \left(i_s+\frac{V}{R}\right)\exp\left(-t\frac{R}{L}\right) - \frac{V}{R} \quad \ldots(105)$$

Therefore

$$i^2 = \left(i_s+\frac{V}{R}\right)^2\exp\left(-2t\frac{R}{L}\right)+\left(\frac{V}{R}\right)^2-\frac{2V}{R}\left(i_s+\frac{V}{R}\right)\exp\left(-t\frac{R}{L}\right) \quad \ldots(106)$$

Integrating then gives

$$\int_0^{t_1} i^2.dt = \left\{\left(i_s+\frac{V}{R}\right)^2\left[-\frac{L}{2R}\exp\left(-2t\frac{R}{L}\right)\right]+\left(\frac{V}{R}\right)^2\left(\frac{V}{R}\right)^2_t+ +\frac{L}{R}.\frac{2V}{R}\left(i_s+\frac{V}{R}\right)\exp\left(-t\frac{R}{L}\right)\right\}_0^{t_1} \quad \ldots(107)$$

Substitution of $i_s = \left(\frac{\sqrt{2}.E_s}{R}.D\right)$ and $V = a\sqrt{2}.E_s$, evaluation between limits, and simplification, gives:

$$I^2_{r.m.s.(diode)} = \left(\frac{\sqrt{2}.E_0}{R}\right)^2\frac{\tan\phi}{\pi}\left[\tfrac{1}{2}-\frac{a}{D}+\left(\frac{a}{D}\right)^2\log_e\left(1+\frac{D}{a}\right)\right]. \quad \ldots(108)$$

This expression is only valid for discontinuous conduction where $\omega t_1 < x_f$.

The armature r.m.s. current is given by Eq.109, which is derived from Eqs.104 and 108.

$$I^2_{r.m.s.(armature)} = I^2_{r.m.s.(thyristor)}+I^2_{r.m.s.(diode)}. \quad \ldots(109)$$

The armature current form factor may be obtained from the equation:

$$\text{Form Factor } F_f = \frac{I_{r.m.s.}}{I_{av}}. \quad \ldots(110)$$

To calculate the increased losses, in a motor armature, caused by the departure of the waveform from pure d.c., the square of the form factor is required. Thus,

$$F_f^2 = \frac{I^2_{r.m.s.\ (armature)}}{I^2_{av\ (armature)}}. \quad \ldots(111)$$

The average armature current can be calculated from the equations given in Appendix 2, while the r.m.s. current equations have been derived above for conduction Types I and II. The form factor may therefore be computed.

The r.m.s. current for conduction type III has not been calculated as the form factor is known to lie between 1·1 and 1·0. The equations for r.m.s. current in this region of operation would be exceedingly complex and the small amount of information to be derived would not normally seem to justify the effort required to evaluate it.

ACKNOWLEDGEMENT

The author wishes to acknowledge the help given by Dr. K. Tweedale and Miss B. Vincent, both of Mullard Research Laboratories, Salfords, for their help with the programming of the computer to evaluate the equations given in this article.

INSTRUCTIONS FOR MOTOR SELECTION

Instructions to enable the selection of a motor, for a given task, are given on pages 316 to 327 These instructions are self-contained, although derived from the main article above, and are accompanied by a worked example.

Instructions for Selecting a D.C. Motor for Thyristor Speed Control

J. Merrett
Mullard Central Application Laboratory

These instructions make possible the selection of a d.c. motor to meet any specific set of requirements of torques and speeds, when thyristor speed control is to be used with a single phase a.c. supply. All relevant graphs are included, and a worked example is appended.

The instructions are based on the article (by the same author, on the same topic) which is published in this issue of Mullard Technical Communications pp 298 to 315).

INTRODUCTION

From the preceding article (described in Ref. 1), it is possible to extract a method of selecting a d.c. motor for thyristor rectified a.c. To facilitate the selection of a motor for such work, this method has been drawn up in a series of instructions which are given below.

The use of these instructions may appear, at first, to be extremely lengthy. When, however, calculations have been performed several times, it will be found that the ease with which each stage of calculations can be carried out increases very considerably.

To clarify the instructions, a worked example is given after the instructions themselves.

INSTRUCTIONS

1 Examine the data describing the work which the motor to be selected is intended to do. Note the exact values of torque, speed and horse-power for several points on this work characteristic, and especially note the torques, speeds and horse-powers, at the extremes of the characteristic. Preferably, include that point which represents maximum power, as this will be needed in section 13.

2 Calculate the horse-power required in the following special way. Multiply the maximum torque at the lowest speed by the maximum speed, and convert this to horse-power.

3 From the catalogue of a motor manufacturer, select a motor such that its output horse-power rating is about 25% more than the horse-power value derived in section 2. This motor constitutes a probable motor for the work to be done, but the following instructions provide full tests of its efficiency under a.c. conditions, and also enable a decision to be made as to whether the motor is in fact suitable.

(For the voltage rating of the motor, or of the armature and field separately, see note 1 on page 327.)

4 Find either the off-load speed of the motor or, preferably, the value of armature current when stalled, each being at rated d.c. voltage. This information may be derived either from the manufacturer, or by measurement. The stalled current, which is the more valuable piece of information of the two, may be calculated by Ohm's law if the d.c. resistance of the armature can be found. Thus, to find the d.c. armature resistance, either by consulting the motor manufacturer or by measurement, usually represents the easiest and most profitable step.

In addition, find out the value of current drawn at rated power on the rated voltage. This can be calculated if the d.c. efficiency at the rated power has been quoted. Calculate, from either of these figures, the rated input power. This will be required in section 13.

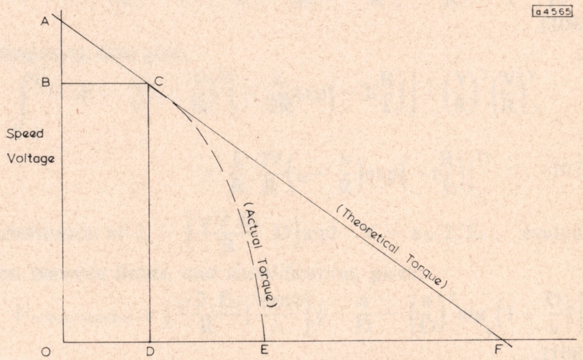


Fig. 1—Sketch of torque and speed data for motor

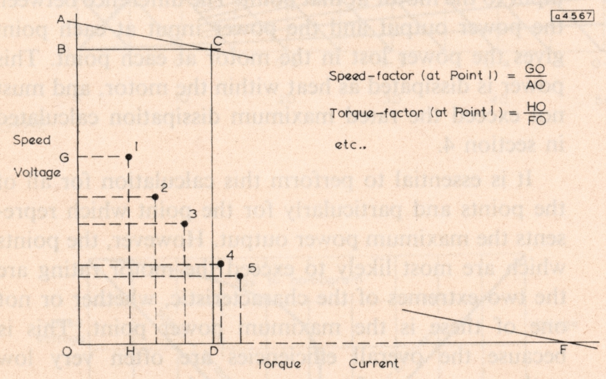
Derive also the rated maximum power dissipation by subtracting the maximum rated output power from the maximum rated input power just evaluated. This dissipation power is required in section 13.

5 Sketch out roughly the information obtained so far, in the way shown in Fig. 1. In this sketch the ordinates are in terms of speed or voltage which are theoretically proportional to each other. The abscissa are in terms of torque or current which also are theoretically proportional to each other.

Insert first of all the point C corresponding to rated maximum horse-power, at the speed B and torque D, all of which are obtainable directly (or by calculation) from the motor catalogue.

Insert secondly either point A or point F, either of which defines the straight line ACF since C has already been inserted. The point A can be inserted if the off-load speed of the motor is known. The point F can be inserted if the stalled current can be derived. The insertion of point F is preferable to the insertion of point A to fully define the line ACF, because the points A and C will nearly always be close together, and the points C and F well separated.

6 Mark the operating points chosen in section 1 on the sketch (Fig. 2) and calculate for each point the fraction 'torque required' divided by 'maximum theoretical torque', and the fraction 'speed required' divided by 'maximum speed'. These are known as the torque factor and speed factor for each point with reference to the motor being considered.



Speed-factor (at Point 1) = $\frac{GO}{AO}$

Torque-factor (at Point 1) = $\frac{HO}{FO}$

etc..

Fig. 2—Torque and speed data for motor and work characteristic

7a Make an initial estimate of the armature efficiency of any one point in the following way. (Treat each point similarly in turn.)

Find the point on Fig. 3 where the relevant torque-factor line and the relevant speed-factor line intersect. This point will fall either on an existing speed-torque line sloping down from the left-hand axis, or on an interpolated line. Note the voltage factor of this speed-torque line, marked at its upper-left extremity. More easily, in practice, the voltage factor of this speed-torque line can be calculated by simply adding the speed factor value to the torque factor value at the point.

Choose a speed-efficiency line sloping down from the right-hand axis such that its voltage factor has the same value as that of the speed-torque line just found. (The selection of this speed-efficiency line will obviously need interpolation if the speed-torque line needed interpolation.)

Find the point where this speed-efficiency line intersects the speed-factor line first used. The required efficiency figure is then the efficiency indicated on the abscissa by this point of intersection.

The efficiency found in this way is approximately correct if it is below 40%, but if it is above 40% it becomes too inaccurate to be usable. Thus, if the efficiency appears to be over 40%, apply the extra step given below.

7b Follow the vertical line up from the efficiency figure already derived to the point where it intersects the $\alpha = 0$ line. From this point of intersection follow the horizontal line to the left until it crosses the α-line of appropriate value. From this new point of intersection follow the vertical line downwards and read off the efficiency on the abscissa. This new reduced efficiency figure is accurate, and is therefore valid under all conditions for the theoretical d.c. performance of the idealised version of the motor being considered, whose torque-speed characteristic has been assumed to be linear in theory as shown in Fig. 1.

If the α-value of the motor being considered is not known, then it may be calculated as the fraction armature current at the off-load speed at rated voltage divided by the stalled current at rated voltage. The motor manufacturer would normally be able to supply the information from his test data.

Repeat sections 6 and 7, to derive the efficiencies of all the points selected on the work characteristic in section 1.

8 At this stage it may be possible to decide, after inspecting the efficiency figure, that the motor being considered (selected in section 3) is not acceptable. This may be evident either because the efficiency figures are unacceptably low, or because they are not sufficiently consistent. Previous experience of calculations based on this series of instructions, or other experience of motor selection for a.c. purposes, may lead to rejecting the motor at this stage for the reasons just mentioned.

If the motor being considered so far is to be rejected, then select a different motor (section 3) and follow the calculation through again to this point, before proceeding further.

(In general, low figures for efficiency at this point in the calculation need not worry the designer, because low efficiencies may not be associated with high heat dissipation. This is because the lowest efficiencies will often occur at low power inputs, particularly if the characteristic is made to approach a constant-torque characteristic. Hence the low efficiency may be associated with only a small heat dissipation. Care should be exercised even here, however, if the motor being considered is cooled by an internal fan: in this case, the heat may become an embarrassment even at low powers if the fan is virtually inoperative because of low speed.)

9 If the operating points being considered have satisfactory efficiency figures with the motor being considered, then 'mark' the operating points on Fig. 4a, 4b or 4c, but it should be noted that the torque factors and speed factors to be marked need slight qualification. That is, 'step-off' the torque-factor from the $\alpha = 0{\cdot}05$ line, or from a line of an appropriate α-value which can be added to the graph; and multiply the speed-factor used in Fig. 3 by $2/\pi$ to make it an a.c. speed-factor before marking it on Fig. 4.

From these points, marked on Fig. 4, read off the appropriate firing-angle of the thyristor, by interpolating between the curves for different firing-angles at the relevant value of cos ϕ. Do this for each point.

The firing angle is affected by the value of L/R for the armature, and therefore the value of cos ϕ. The curves for cos $\phi = 0{\cdot}2$ and cos $\phi = 0{\cdot}4$ given in the graphs will allow a sufficiently accurate estimate to be made by interpolation, or by a limited amount of extrapolation should this ever be required. The value of cos ϕ can usually be obtained from the manufacturer, or it can be measured directly.

10 Read-off from Fig. 5a or Fig. 5b the appropriate value of the square of the form-factor, using the firing-angle just found in section 9 and the a.c. speed-factor calculated in section 9. Do this for each point.

11 Calculate the final efficiency on a.c. for each operating point in the the following way.

Calculate the armature copper loss: this equals 100% minus the d.c. efficiency found in section 7a. Multiply the copper loss by the square of the form factor (found in section 9) to give the a.c. copper loss. Add to this a.c. copper loss the iron, mechanical, etc. losses, which are represented by the difference between the efficiency for $\alpha = 0$ and the efficiency for the relevant α-value. Subtract this total loss figure from 100% to give the new reduced a.c. efficiency.

Add this increased value of armature copper loss to the iron etc. loss, and subtract from 100% to yield the new reduced efficiency figure.

At this stage the calculation may come to a halt because the apparent final efficiency at a point becomes less than zero. A different motor in section 3 is then obviously required. (Read, however, Note 1 on page 327.)

12 Again, at this stage, examine the set of final efficiency figures derived for the points on the operating curve. If they are obviously too low, or too inconsistent, then the working region must be changed. This can be effected only by again 'replacing' the catalogue motor, with a different catalogue motor, and following the calculation through again to this point: at this stage, however, if the figures are not far out, the same motor with a different d.c. voltage rating for the armature will often provide suitable figures (see Note 1 on page 327).

13 If the set of efficiency figures derived give the appearance of being satisfactory, the final calculation is then to establish whether or not the motor selected (section 3) has a suitable horse-power rating. This is because the de-rating procedure followed in sections 2 and 3 may have been insufficient, in which case a larger motor must be selected, or it may have been unnecessarily cautious, in which case the choice of a smaller motor is possible. Make the final calculation as follows.

The efficiency on a.c. has already been found for each point (section 11), and the power required at each point was stated in section 1. Multiply the power required at each point by 100% divided by the percentage efficiency at that point. This gives the power input to the motor at that point. The difference between the power output and the power input at each point gives the power lost in the motor at each point. This power is dissipated as heat within the motor, and must not exceed the rated maximum dissipation calculated in section 4.

It is essential to perform this calculation for all of the points and particularly for the point which represents the maximum power output. However, the points which are most likely to exceed the motor rating are the two extremes of the characteristic, whether or not one of these is the maximum power point. This is because the overall efficiencies are often very low at these extremes of torque or speed.

14 If the final horse-power loss derived for any point exceeds the maximum horse-power loss rating (section 4) of the motor selected, then select a larger motor and repeat the calculations for the same purpose, that is, to check that this motor is in fact of a suitable power rating.

If the final horse-power loss figure derived falls far short of the maximum horse-power loss rating (section 4) of the motor selected, there may be a smaller motor available which will meet the requirement.

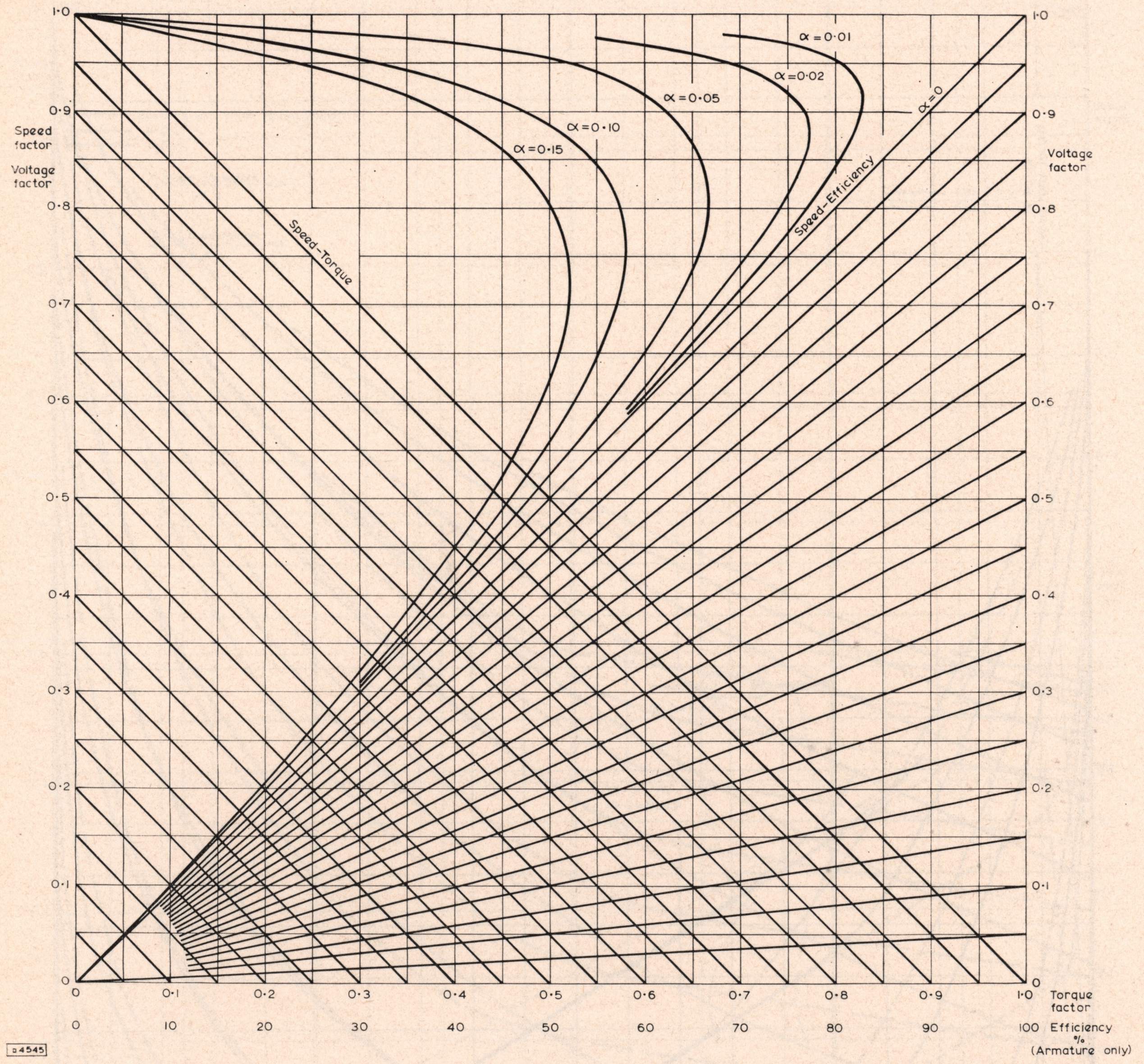
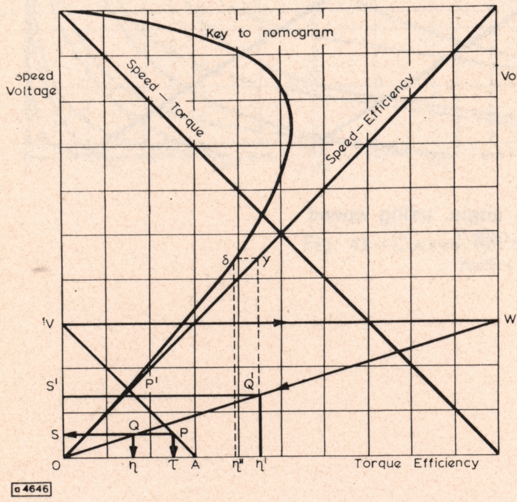


Fig. 3—Nomogram relating speed, torque and efficiency for d.c. shunt motor with armature voltage speed-control



To find motor efficiency when operating speed and torque are known

1. Select a speed-torque line which passes through the desired operating point P corresponding with speed S and torque T, line VPA.
2. Select a speed-efficiency line WQO having the same voltage-factor as line VPA. The intersection of this line with the speed line SQP, gives the required motor efficiency η.
3. The above method gives η for $\alpha = 0$. This is approximately correct if $\eta < 40\%$. For higher values of η, the efficiency can be found in the following way.
First evaluate the efficiency for $\alpha = 0$, as already explained. Find the point of intersection between this efficiency line (say η^1) and the speed-efficiency line for $\alpha = 0$. (This would give the point γ.) Draw a horizontal line through this point to intersect the speed-efficiency curve for the correct value of α. (This would give the point δ.) The new efficiency (η'') can then be read off from the abscissa below this point (V).

N.B.—Air gap torque at no load is assumed to be proportional to speed, and includes all losses (bearing, windage, iron, etc.) except copper losses. Field losses and armature reaction are not taken into account.

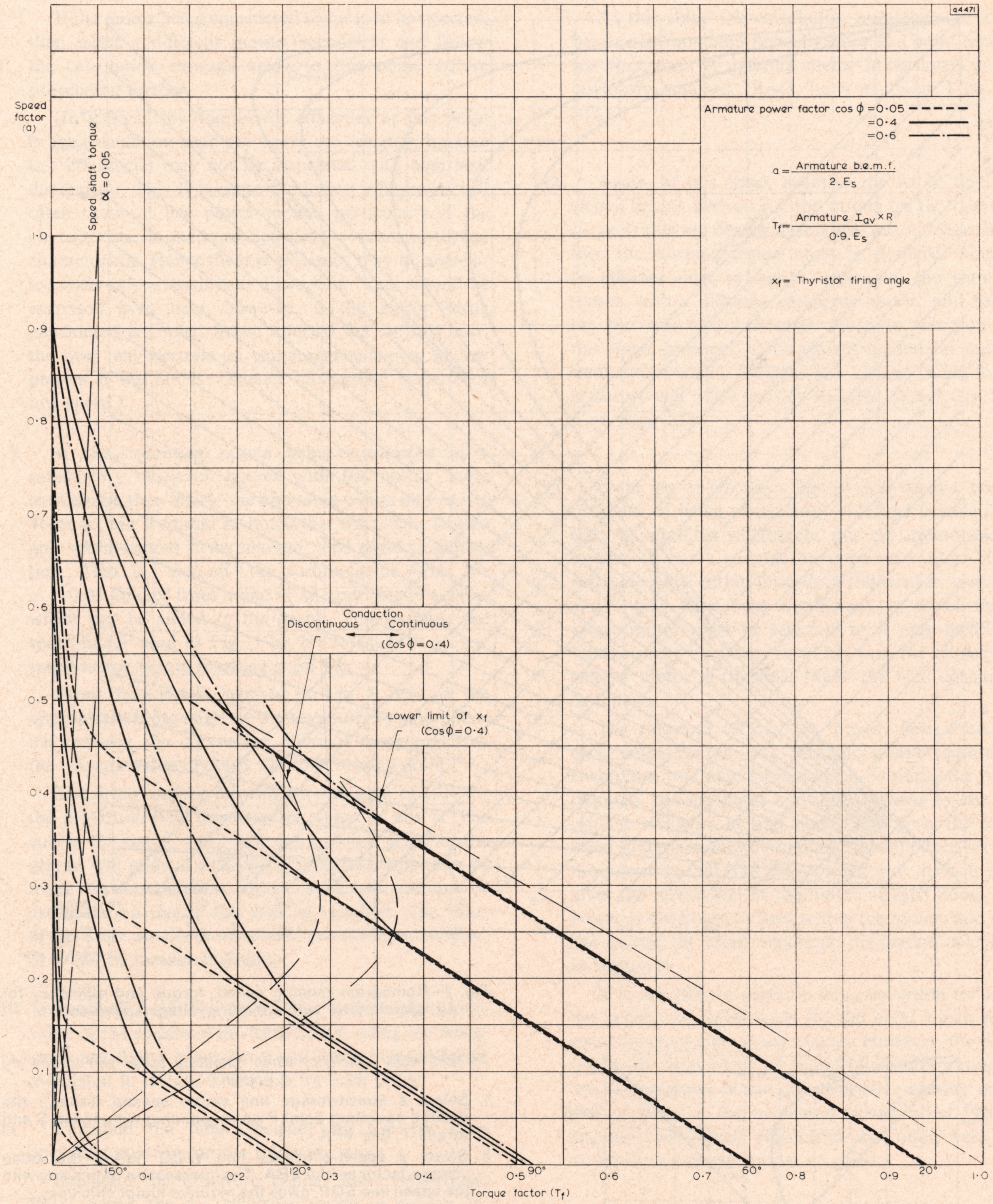


Fig. 4a—Curves for derivation of firing angle, using speed factor, torque factor and cos ϕ

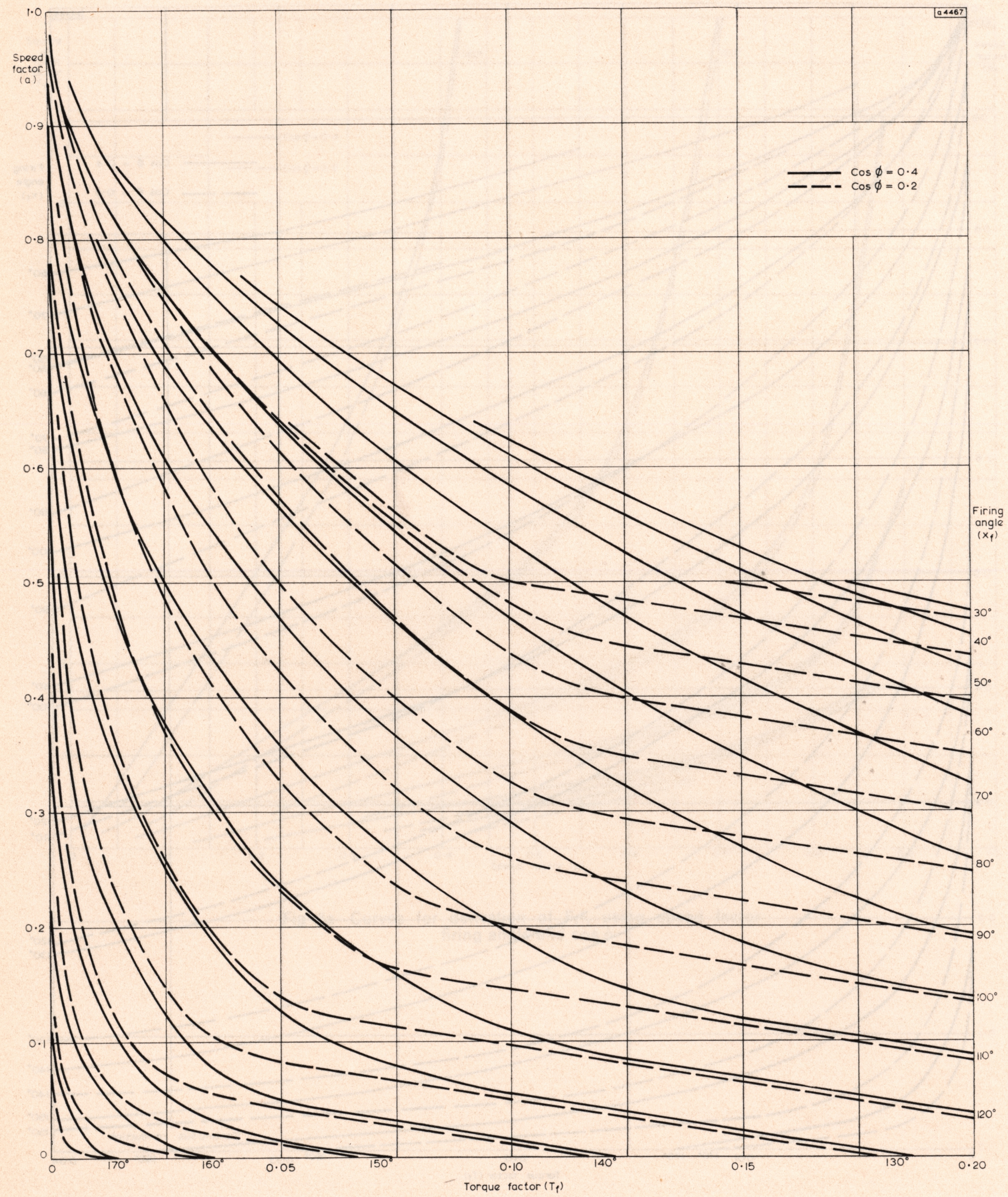


Fig. 4b—Curves for derivation of firing angle, using speed factor, torque factor and cos ϕ

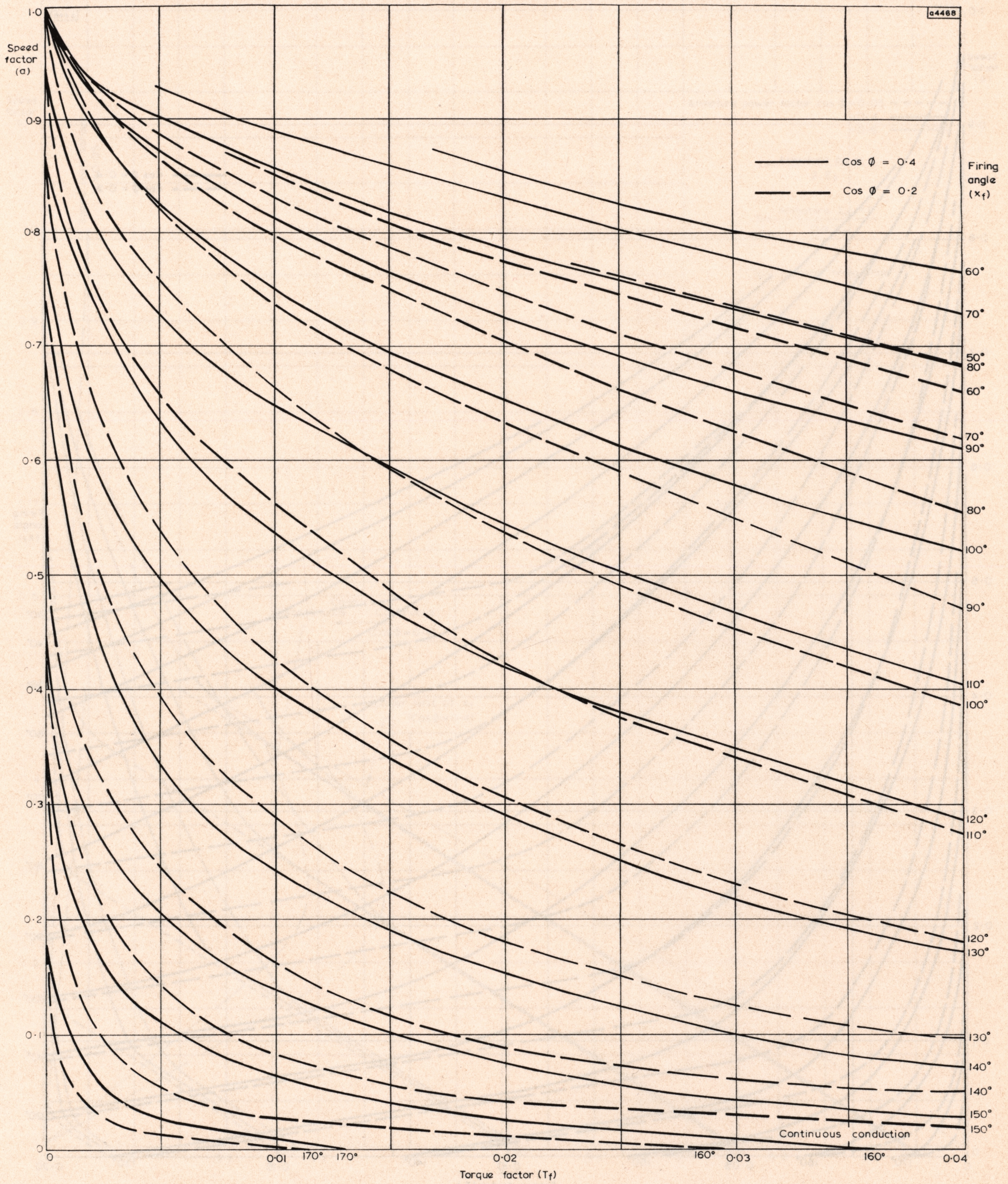


Fig. 4c—Curves for derivation of firing angle, using speed factor, torque factor and cos ϕ

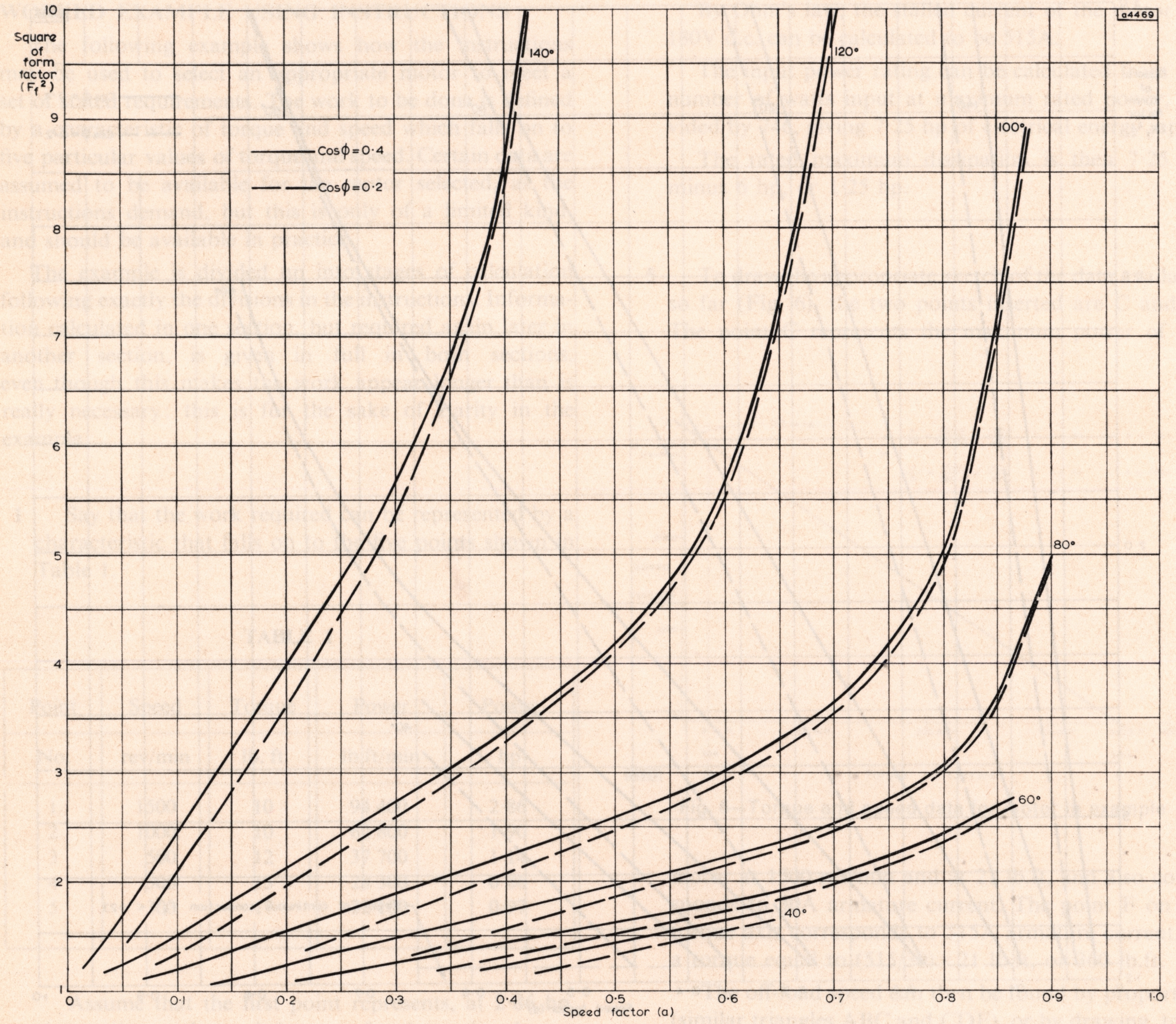


Fig 5a—Curves for derivation of F_f^2, using speed factor, firing angle and cos ϕ

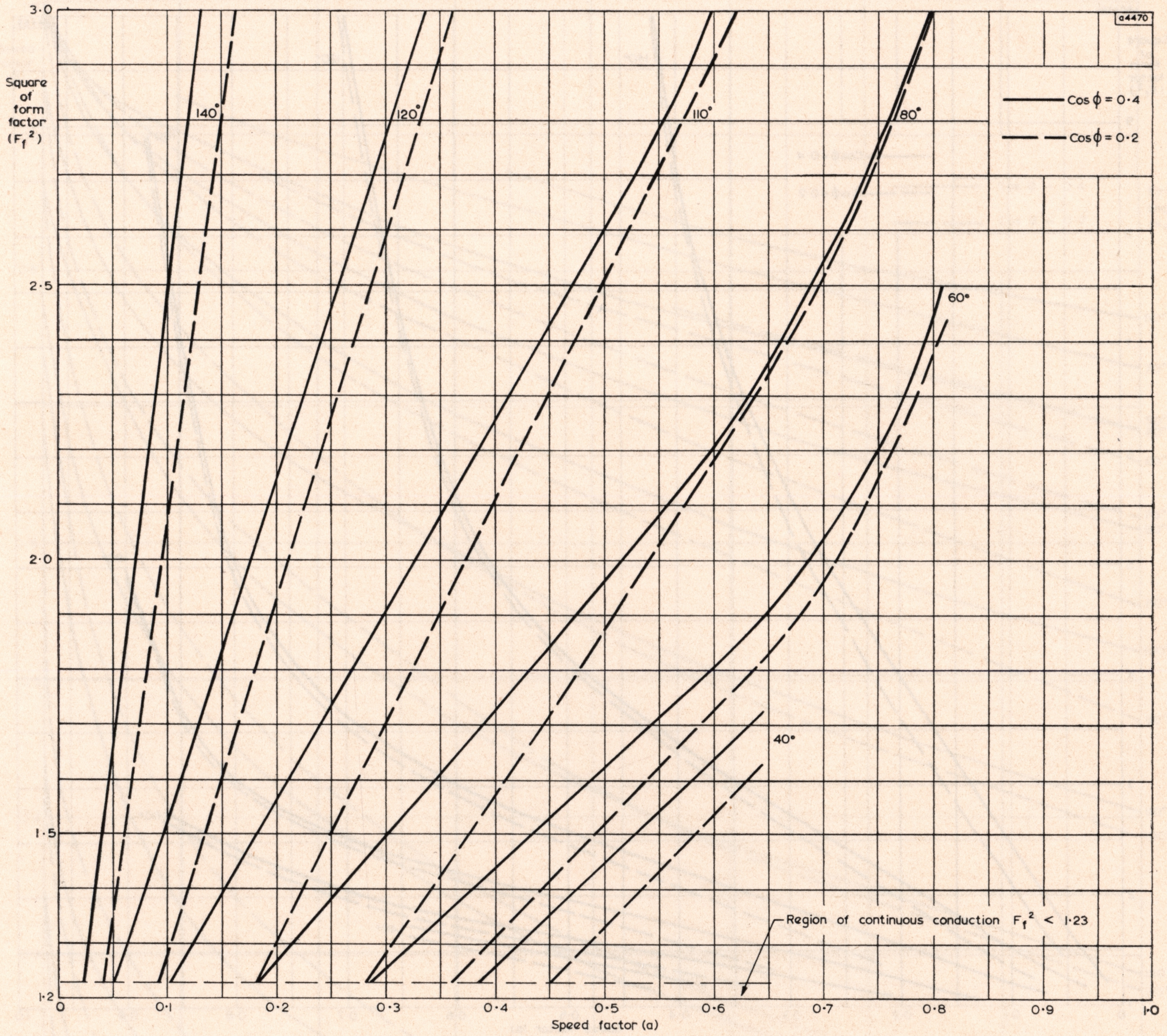


Fig. 5b—Curves for derivation of F_f^2, using speed factor, firing angle and cos ϕ

WORKED EXAMPLE USING INSTRUCTIONS

The following example shows how the instructions may be used to select an appropriate motor to meet a set of stated requirements. The work to be done is defined by a characteristic of torque and speed which falls on to five particular values of torque and speed. Certain data are assumed to be available for the motor selected, as the instructions demand, but this is only of a limited kind, and should be available in practice.

The example is divided up into stages of calculation following exactly the divisions in the instructions. Information calculated in one section, but required again later in another section, is given in full in both sections, even though this makes the work appear longer than is really necessary: this is for the sake of clarity in the example.

1 Say that the work required can be represented by a characteristic that falls on to the five points shown in Table 1.

TABLE 1

Point No.	Speed rev/min	Torque lb. ft	Power lb.ft/min	Power hp
1	1500	10	94 400	2·86
2	1000	10	62 800	1·90
3	500	12	37 700	1·14
4	300	15	28 300	0·86
5	300	17	32 100	0·97

Assume that the first point represents, at 2·86 hp, the maximum demanded power for the entire characteristic, and that the first and fifth points represent the extremes of the characteristic.

2 The power required is then $2\pi \times 17$ lb.ft × 1500 rev/min. This equals 160 000 lb.ft/min, or 4·85 hp.

3 The need for a motor capable of delivering 4·85 hp increased by about 25% would probably lead to the selection of a 6 hp. motor from a motor manufacturer's catalogue.

Assume then that the remaining data for the motor are that the 6 hp is stated to beat 1500 rev/min (and therefore, by calculation, to be at 21 lb.ft) and that the armature is intended for 180V d.c.

(See Note 1 on page 327.)

4 Assume that the d.c. armature resistance, brush-to-brush, has been found to be 0·35Ω, and that the armature current at peak power at 1500 rev/min on d.c. has been found to be 30A. Assume, also, that the off-load speed has not been quoted.

By Ohm's law, the stalled current of the motor on 180V d.c. can be calculated to be 515A.

The input power rating can be calculated from the number of watts input at maximum rated power, divided by 746, giving 7·25 hp of electrical energy input.

The rated maximum dissipation is then 7·25 hp minus 6 hp, or 1·25 hp.

5 To draw the appropriate sketch of the data available so far (Fig. 6), the two points inserted are C and F. The point C represents the maximum power of the motor at 1500 rev/min and at 21 lb.ft, and also corresponds to 30A armature current. The point F on the torque axis corresponds to 515A armature current, or a torque equal to (515/30)×21 lb.ft, or 360 lb.ft.

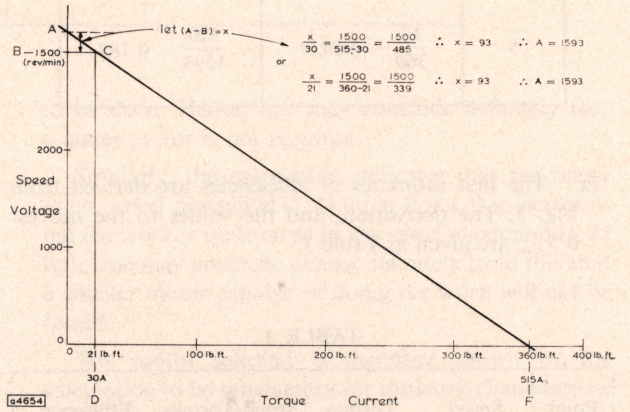


Fig. 6—Torque and speed data for motor in example

The off-load speed can then be found by proportion (similar triangles ABC and CDF), or by drawing, to be 1593 rev/min.

6 The five operating points are now marked on the sketch, redrawn in Fig. 7.

The torque factor and speed factor for each of the five points, are calculated in the way shown in Table 2.

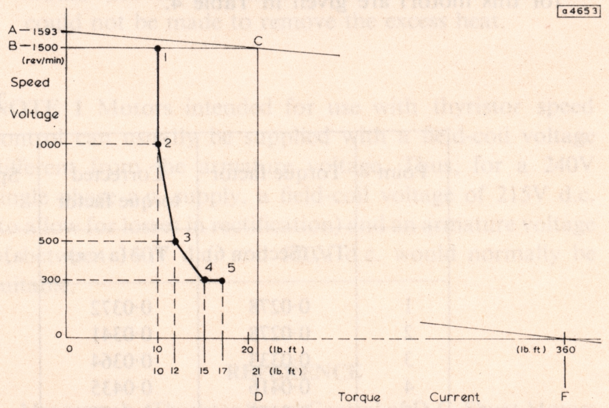


Fig. 7—Torque and speed data for motor and work characteristic in example

TABLE 2

Point	Torque factor	Speed factor
1	$\frac{10}{360} = 0{\cdot}0278$	$\frac{1500}{1593} = 0{\cdot}941$
2	$\frac{10}{360} = 0{\cdot}0278$	$\frac{1000}{1593} = 0{\cdot}627$
3	$\frac{12}{360} = 0{\cdot}0333$	$\frac{500}{1593} = 0{\cdot}314$
4	$\frac{15}{360} = 0{\cdot}0416$	$\frac{300}{1593} = 0{\cdot}188$
5	$\frac{17}{360} = 0{\cdot}0472$	$\frac{300}{1593} = 0{\cdot}188$

7a The first estimates of efficiencies are derived from Fig. 3. The derivation, and the values to the nearest 0·5%, are given in Table 3.

TABLE 3

Point	Speed factor	Torque factor	Speed-Torque voltage-factor	Efficiency 1st estimate
	Section 6	Section 6		Fig. 7
No.	a	T_f	$a+T_f$	%
1	0·941	0·0278	0·969	97·0
2	0·627	0·0278	0·655	96·0
3	0·314	0·0333	0·347	90·5
4	0·188	0·0416	0·230	81·5
5	0·188	0·0472	0·235	80·5

7b Since most of these points have an apparent efficiency of 40% or more, the correction for the α-value is applied. The corrected values of efficiency for $\alpha = 0{\cdot}01$ (assumed for this motor) are given in Table 4.

TABLE 4

Point	D.C. efficiency	D.C. efficiency
No.	1st Estimate. (α=0)	2nd Estimate. (α=0·01)
1	97·0%	73·5%
2	96·0%	77·5%
3	90·5%	82·5%
4	81·5%	78·0%
5	80·5%	77·0%

8 Assume that all of these efficiencies are judged to be sufficiently high and sufficiently consistent. Therefore we proceed with the calculation.

9 The five points are 'marked' on Fig. 4a, or 4b, or 4c, using the qualifications indicated in instruction 9, and with $\alpha = 0{\cdot}01$. The factors 'marked-off' are listed, in Table 5, together with the resulting firing angles for $\cos\phi = 0{\cdot}2$.

10 The squares of the form factors are read-off from Fig. 5 for $\cos\phi = 0{\cdot}2$, and are given in Table 6.

TABLE 6

Point	A.C. speed factor	Firing angle	Figure used	F_f^2
	Section 9	Section 9		
1	0·592	77°	5b	2·08
2	0·394	103°	5b	2·21
3	0·197	120°	5b	1·90
4	0·118	126°	5b	1·54
5	0·118	124°	5b	1·49

TABLE 5

Point	Torque factor	Corrected torque factor	Speed factor	Corrected speed factor	Fig. used	Firing angle ($\cos\phi = 0{\cdot}2$)
	T_f, (Section 6)	$T_f+(a\times\alpha)$	a, (Section 6)	$a\times 2/\pi$		
1	0·0278	0·0372	0·941	0·592	4c	77°
2	0·0278	0·0341	0·627	0·394	4c	103°
3	0·0333	0·0364	0·314	0·197	4c	120°
4	0·0416	0·0435	0·188	0·118	4b	126°
5	0·0472	0·0491	0·188	0·118	4b	124°

TABLE 7

Point	D.C. efficiency	Copper loss	Iron etc. losses	F_f^2	A.C. copper losses	A.C. efficiency
	(*A*)	(*B*)	(*C*)	(*D*)	(*E*)	(*F*)
	$\eta_{(\alpha\,=\,0\cdot01)}$ Section 7	$100-\eta_{(\alpha\,=\,0)}$	$100-(A)-(B)$	Section (10)	$(B)\times(D)$	$100-(C)-(E)$
1	73·5%	3·0%	23·5%	2·08	6·5%	70·0%
2	77·5%	4·0%	18·5%	2·21	9·0%	72·5%
3	82·5%	9·5%	8·0%	1·90	18·0%	74·0%
4	78·0%	18·5%	3·5%	1·54	28·5%	68·0%
5	77·0%	19·5%	3·5%	1·49	29·0%	67·5%

11 The final efficiencies have been calculated as shown in Table 7. Figures derived at various steps in the calculation are given, together with a brief re-statement of the method of calculation. The efficiency figures and loss figure are shown in Table 7 and are again given to the nearest 0·5%.

Note on sections 11 and 13. The letters (A), (B), (C), etc. are used in these tables merely to label the content of each coloumn: this enables the instructions for calculations in other columns to be shown easily in the line below the letters. These bracketed letters have no association with the unbracketed letters used earlier in the calculation in connection with Figs. 1, 2, 6 and 7.

12 Assume that, once again, these final efficiency figures are judged to be potentially satisfactory. We then proceed with the final calculation.

13 The input powers demanded by the motor at the various points of the characteristic are calculated as shown in Table 8.

14 At each point on the characteristic the dissipated power, given in column (*J*) in Table 8, is less than the rated maximum dissipated power (1·25 hp) calculated in section 4.

Therefore the motor being considered has been proved by the calculation to be adequate for the work to be done. Hence, one may conclude definitely that a larger motor is not required.

TABLE 8

Point	Power required	Final efficiency	Power input demanded	Power dissipated in motor
	(*G*)	(*H*)	(*I*)	(*J*)
	Section 1	Section 11	$(G)/(H)$	$(I)-(G)$
1	2·86 hp	70·0	4·09 hp	1·23 hp
2	1·90 hp	72·5	2·62 hp	0·72 hp
3	1·14 hp	74·0	1·54 hp	0·40 hp
4	0·86 hp	68·0	1·26 hp	0·40 hp
5	0·97 hp	67·5	1·44 hp	0·47 hp

Similarly, the calculation indicates that the maximum power dissipated (1·23 hp at Point 1) in performing the work is quite close to the rated maximum (1·25 hp). One may conclude almost definitely from this that a smaller motor capable of doing the work will not be found.

The motor selected is therefore shown by the calculation to be satisfactory for the work characteristic which is to be met.

It is worthwhile noting that the dissipated power approaches most closely to the rated maximum in this example at the highest speed of the characteristic, and that this speed is also at the rated speed of the motor. Therefore it is possible that, had the calculations indicated a very slight exceeding of the rated dissipation at Point 1 this could have been regarded as permissible since the fan is usually slightly overrated at high speed, or alternatively the excess could be made permissible by a slight modification to the fan, a facility offered by some motor manufacturers. For example, an excess of 0·2 hp at the highest speed of the motor would demand the removal of approximately an extra 150W, or about 12% of the rated dissipation.

Such an excess power at the lower speeds, however, would not be permissible because the cooling fan could not be made to remove the excess heat.

NOTE 1 Motors intended for use with thyristor speed control can usually be supplied with a field-coil voltage different from the armature voltage. Thus, for a 240V single phase a.c. supply, a field-coil voltage of 215V d.c. (to allow for losses in rectification) and an armature voltage of between 160V d.c. and 180V d.c. would normally be suitable.

REFERENCE

1. MERRETT, J., 'Thyristor Speed Control of D.C. Shunt Motors from a Single Phase Supply', **Mullard Technical Communications,** Vol. 8, No. 80, pp 298 to 315.

Articles on Power Topics

The following publications on power topics are available, free of charge:

	Reference
Power Control for Industry	TP840
Power Rectification with Silicon Diodes	TP552
Current Overload Protection in Thyristor Phase Control Systems	Ch
Regenerative Braking of D.C. Motors	Cg
Transient Performance of Heatsinks for Power Devices	Cd
Parallel Operation of Silicon Rectifier Diodes	Cc
Rectifier Diode Operation at Kilocycle Frequencies	Cb
Thermal Resistance of Semiconductor Devices under Steady-state Conditions	Bs
Transistorised Circuits for SCR Firing	Bp
Firing Requirements for Silicon Controlled Rectifiers*	Bf
Mains Overvoltages: Protection and Monitoring Circuits	Bc
Design of Cooling Fins for Silicon Power Rectifiers	Ak

* * *

"Power Control for Industry" is concerned with rectifier stacks. The other items in the list are reprints of articles which have appeared in *Mullard Technical Communications.*

Data sheets for Rectifier Diodes, Thyristors, and Stacks will be found in Volume 4 of the **Mullard Technical Handbook.** The Handbook is available on a subscription basis.

**temporarily out of print*